Effects of Topology and Mobility in Bio-Inspired Synchronization of Mobile Ad Hoc Networks

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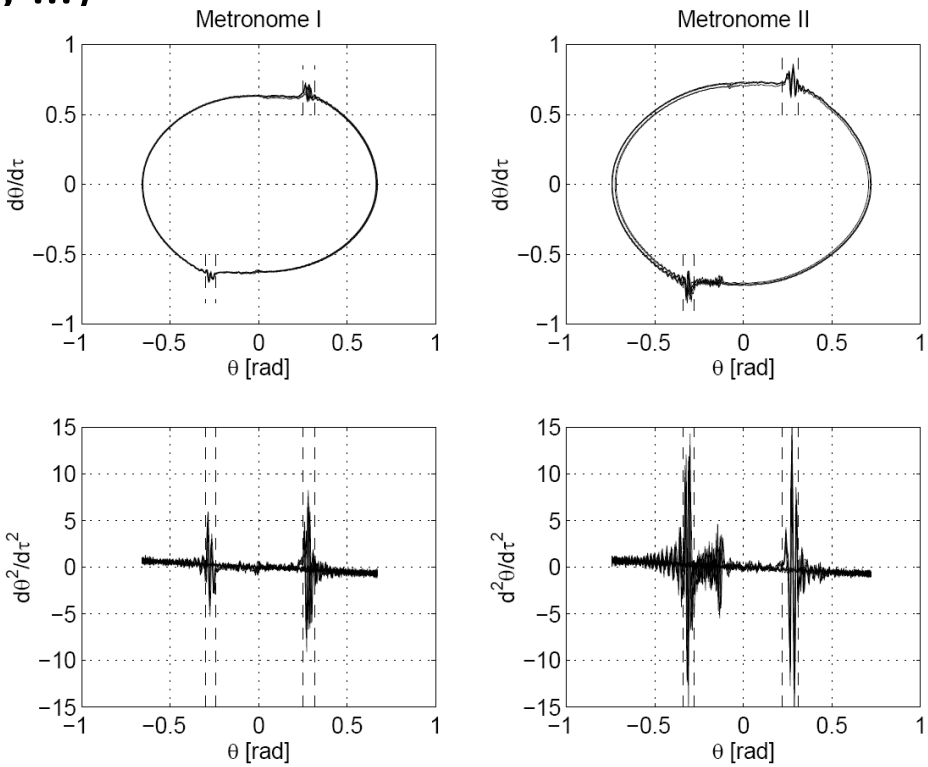
Spontaneous Synchronization

- Spontaneous synchronization occurs ubiquitously in nature (from particles, to atoms, to molecules, to cells, to insects, to humans, to planets, to galaxies, …)

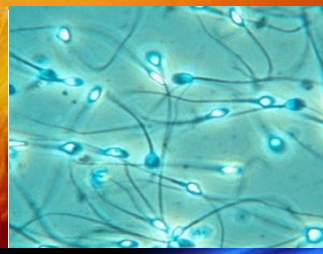


W.T. Oud, H. Nijmeijer and A.Yu. Pogromsky "A Study of Huijgens' Synchronization: Experimental Results", Lecture Notes in Control and Information Sciences, Springer, 2006







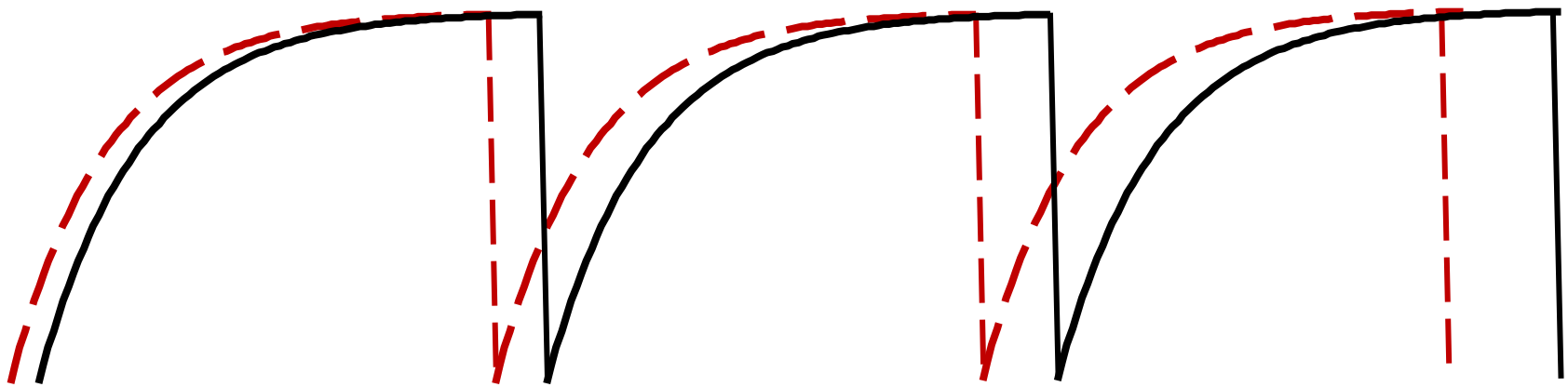





Basic model

R. Mirollo and S. Strogatz, “Synchronization of pulse-coupled biological oscillators”, SIAM Journal of Applied Mathematics, December 1990

Each oscillator follows a simple law of “Integrate and fire”



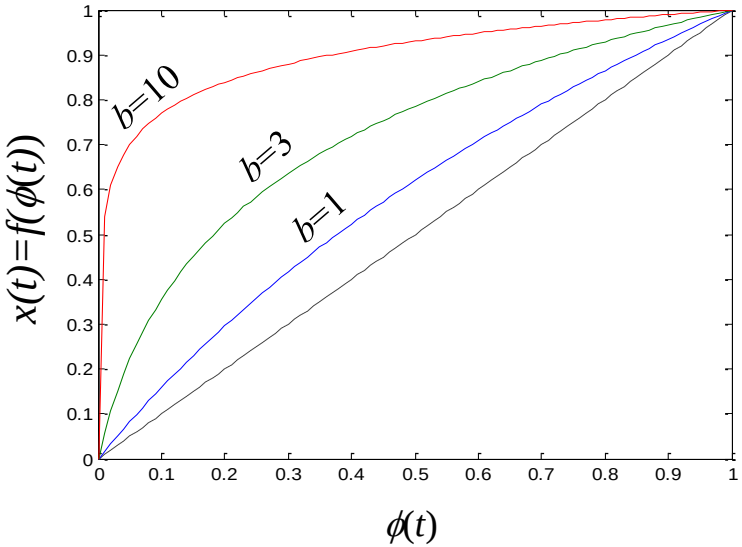
Then they interact through a simple rule: When an oscillator fires, neighbors state advances a small quantity ε

$$x_i(t) = 1 \Rightarrow \begin{cases} x_i(t^+) = 0 \\ x_j(t^+) = \min(1, x_j(t^-) + \varepsilon) \quad j \text{ neighbor of } i, j \neq i \end{cases}$$

Charging functions

At instant t, an oscilador is in a fraction ϕ of its phase,

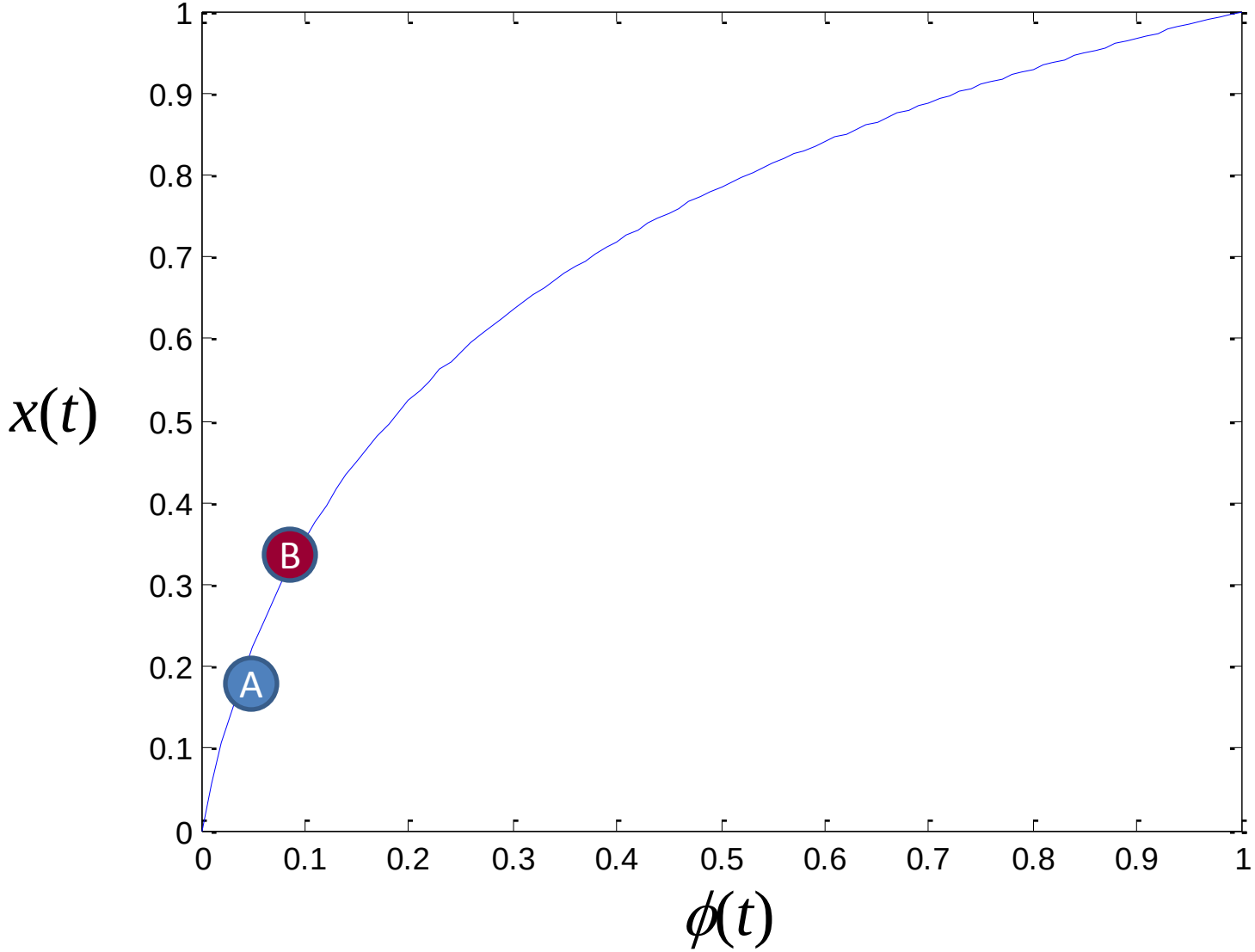
$$x(t) = f(\phi(t)) = \frac{1}{b}\log\left(1+\left(e^{b}-1\right)\phi\right)$$



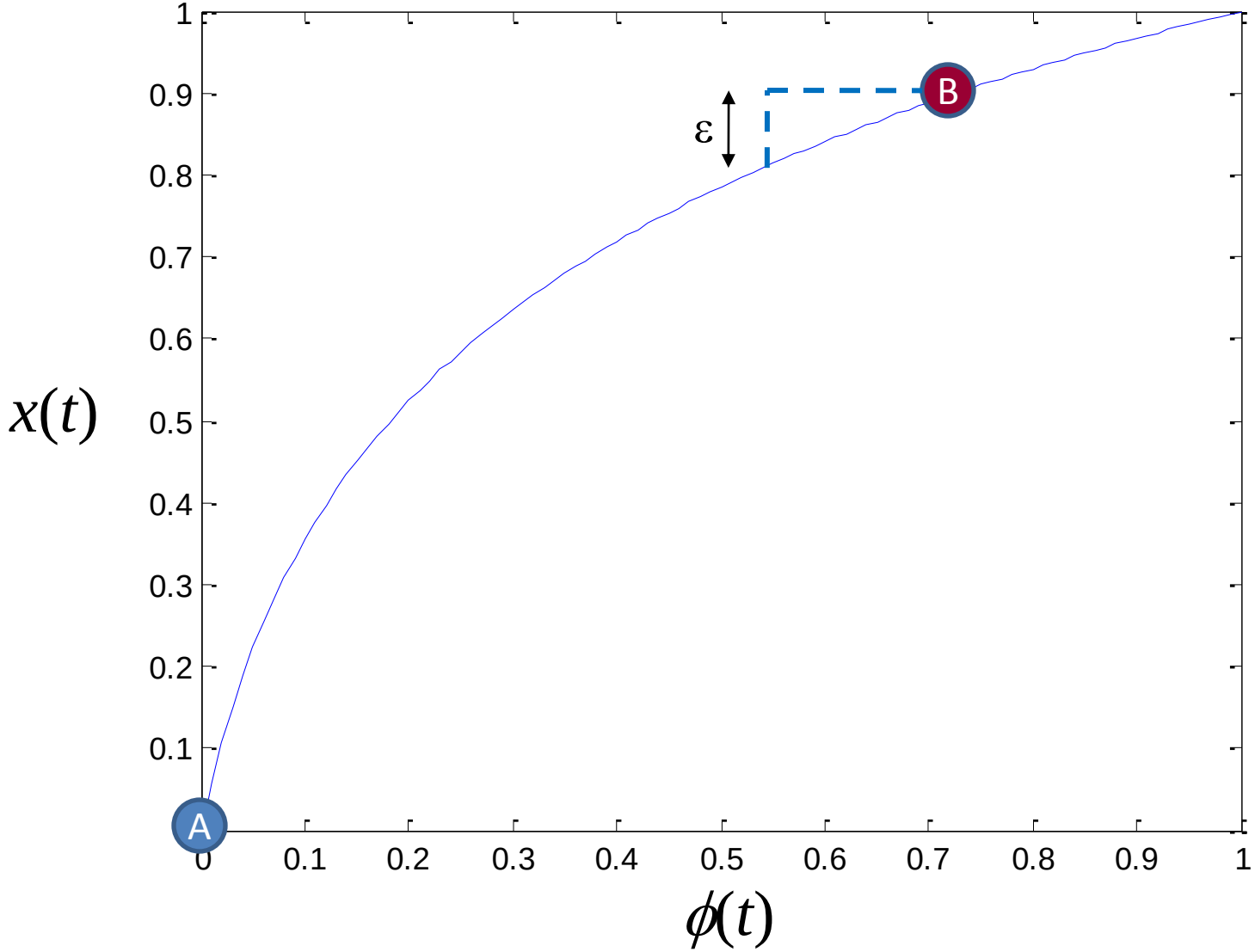
- When the oscillator is in state $x(t)$, it is in a fraction ϕ of its phase,

$$\phi(t) = g(x(t)) = g\left(f(\phi(t))\right) = \frac{e^{bx}-1}{e^{b}-1}$$

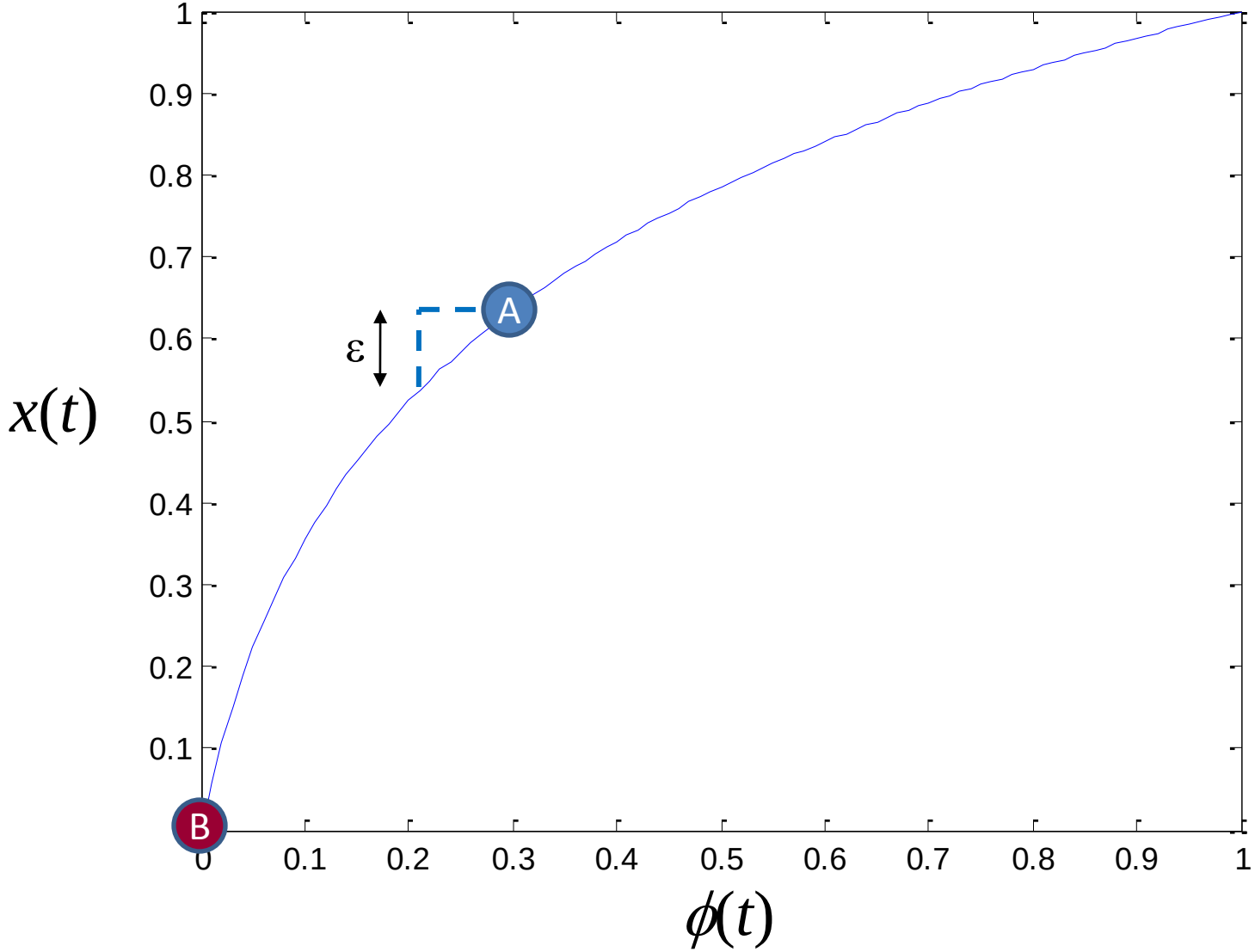
Two coupled oscillators



Two coupled oscillators

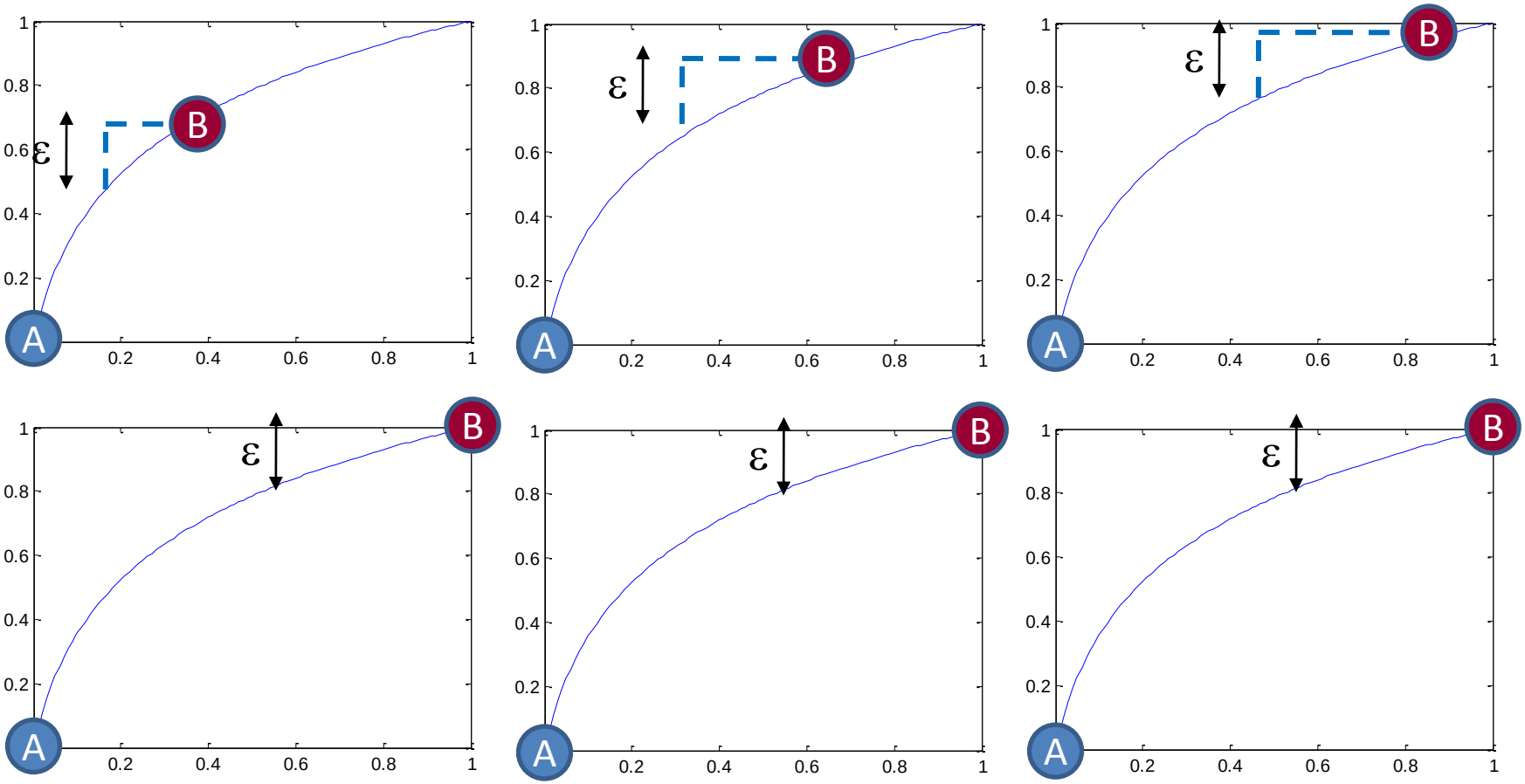


Two coupled oscillators



Two coupled oscillators

Let us put an stroboscope immediately after A fires



Two coupled oscillators

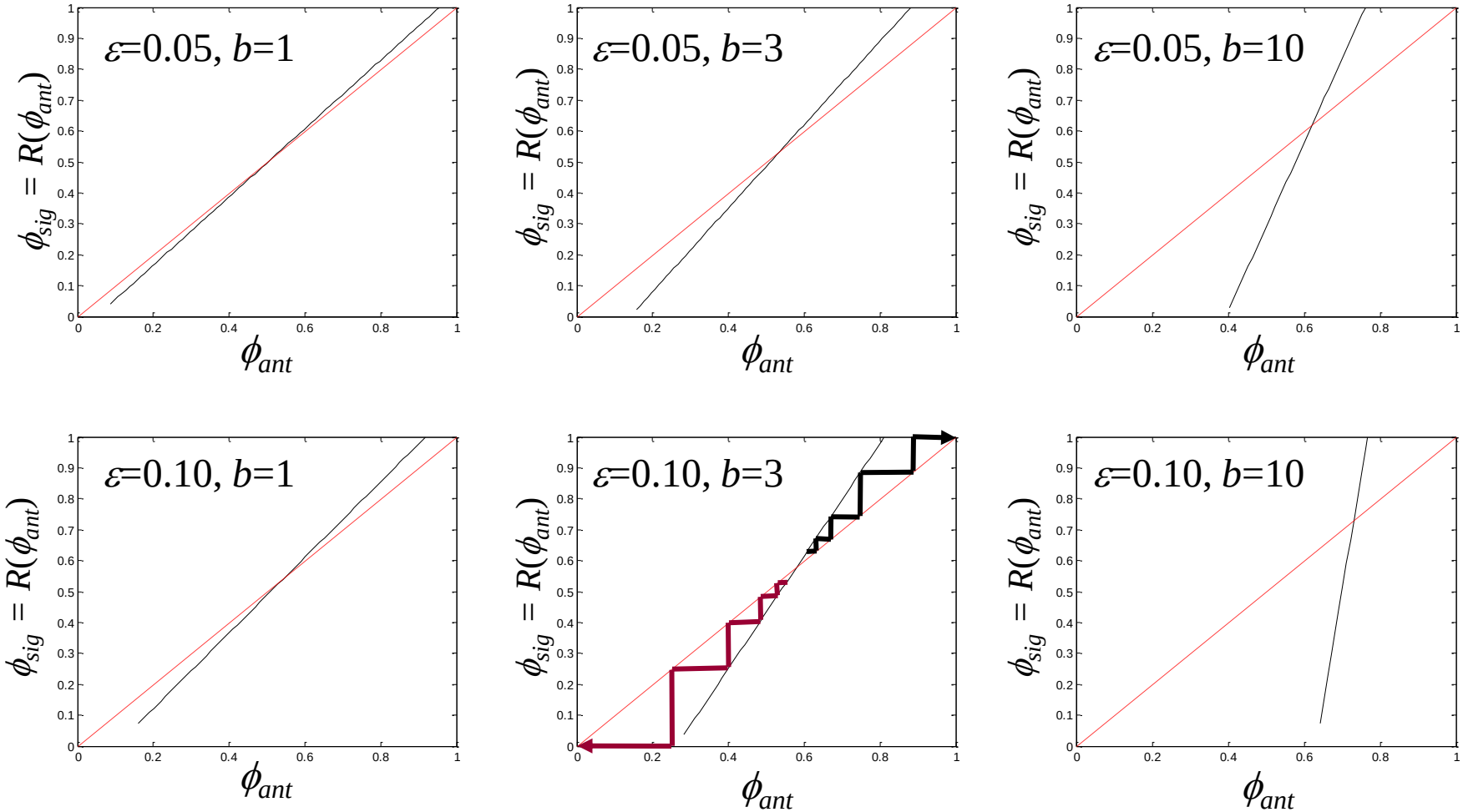
- After A fires, A's phase is zero and B's phase is ϕ
- B will fire when its phase advances $1-\phi$. By then, A will be in $x_A = f(1-\phi)$
- Then, B returns to zero and A jumps to $x_A = \min(1, \varepsilon + f(1-\phi))$
- If $x_A = 1$, we achieved synchronism.
- otherwise, $x_A = \varepsilon + f(1-\phi) < 1$ and A's phase is $h(\phi) = g(\varepsilon + f(1-\phi))$
- In the next round, if they have not achieves synch, their phases will be $(0, h(h(\phi)))$
- **Return map**: phase of B the next time A fires, if it was at phase ϕ at the previous firing of A

$$R(\phi) = h\big(h(\phi)\big) = g\Big(\varepsilon + f\Big(1 - g\big(\varepsilon + f(1-\phi)\big)\Big)\Big)$$

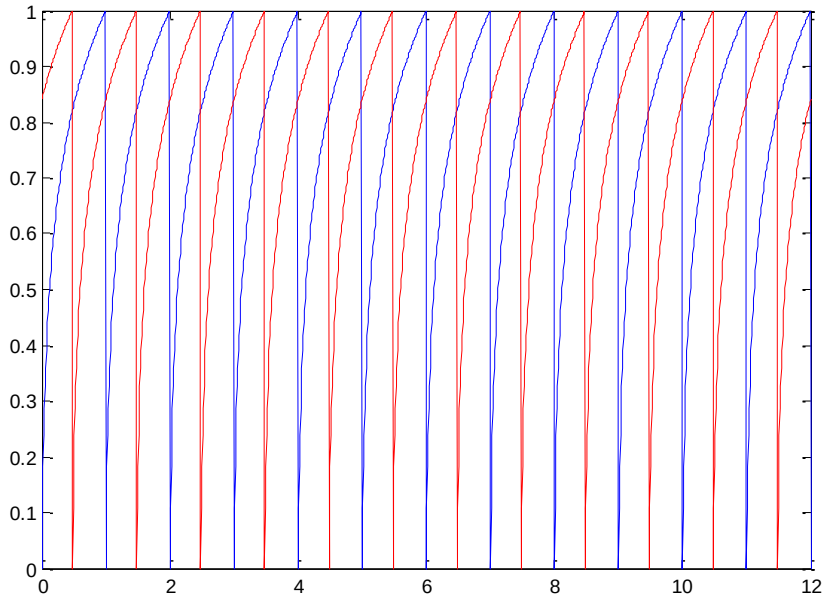
Two coupled oscillators

The return map has a unique fixed point, $\phi^* = \dfrac{e^{b(1+\varepsilon)} - 1}{\left(e^{b} - 1\right)\left(e^{b\varepsilon} - 1\right)}$

Which is a repeller

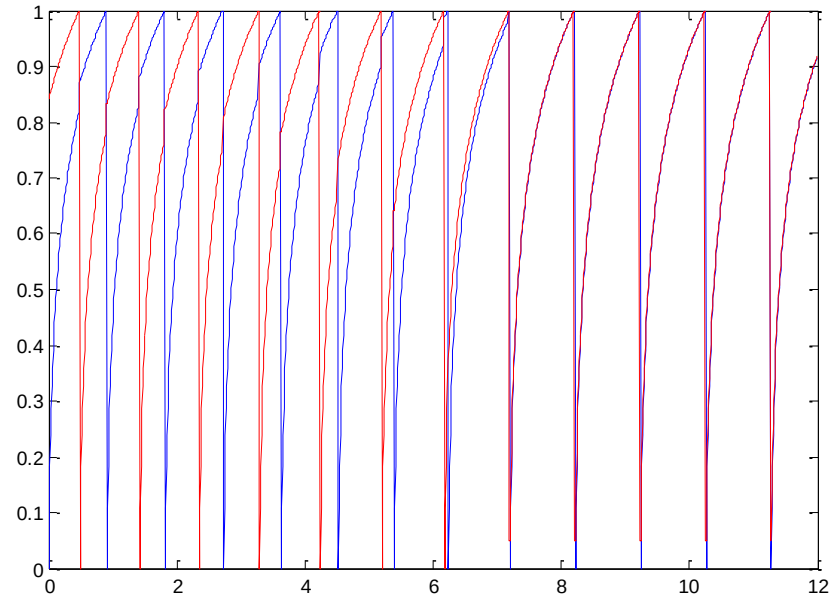


Synchronization is unavoidable



Independent Oscillators

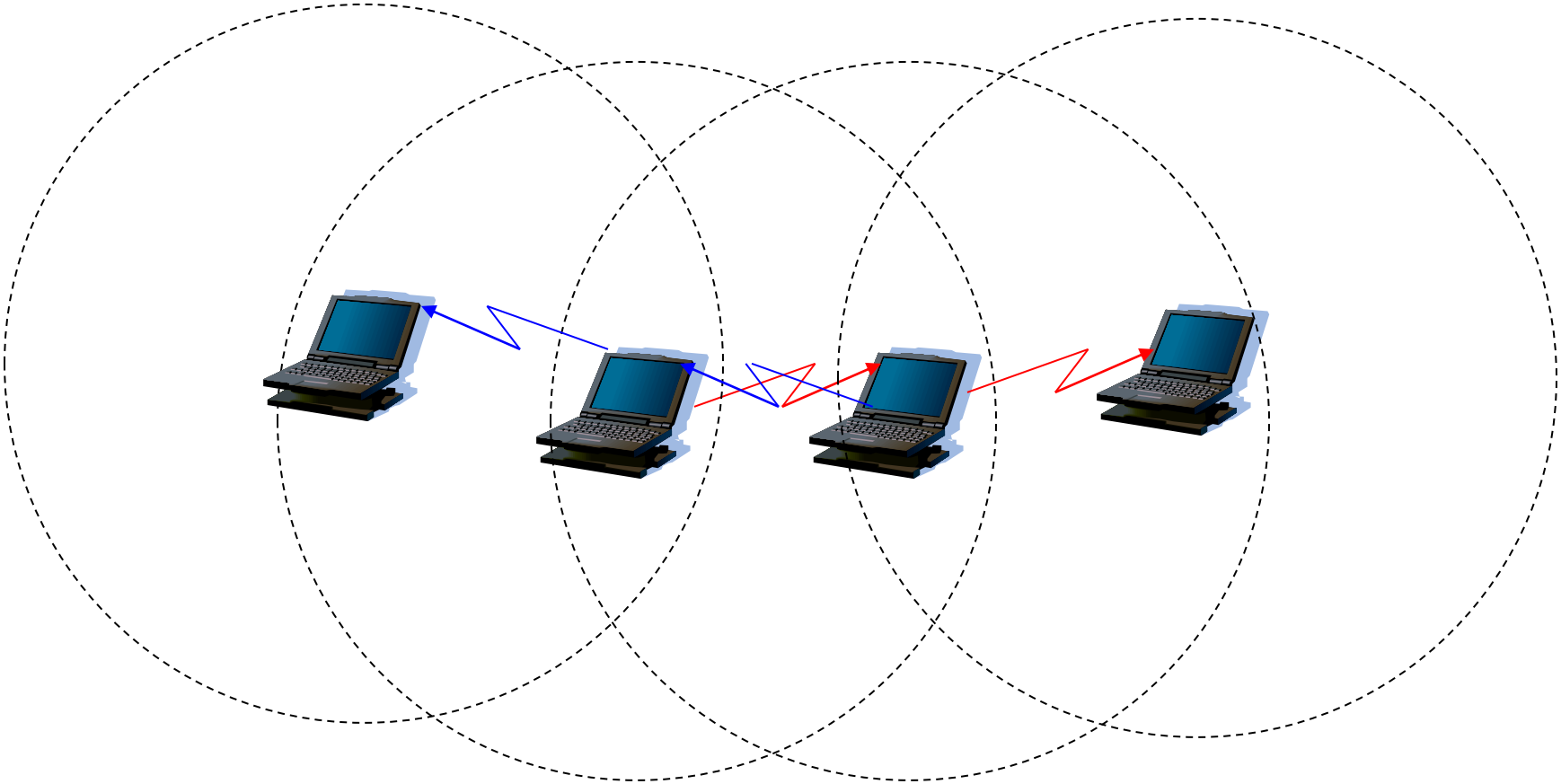
$b = 3,\ \varepsilon = 0.0$



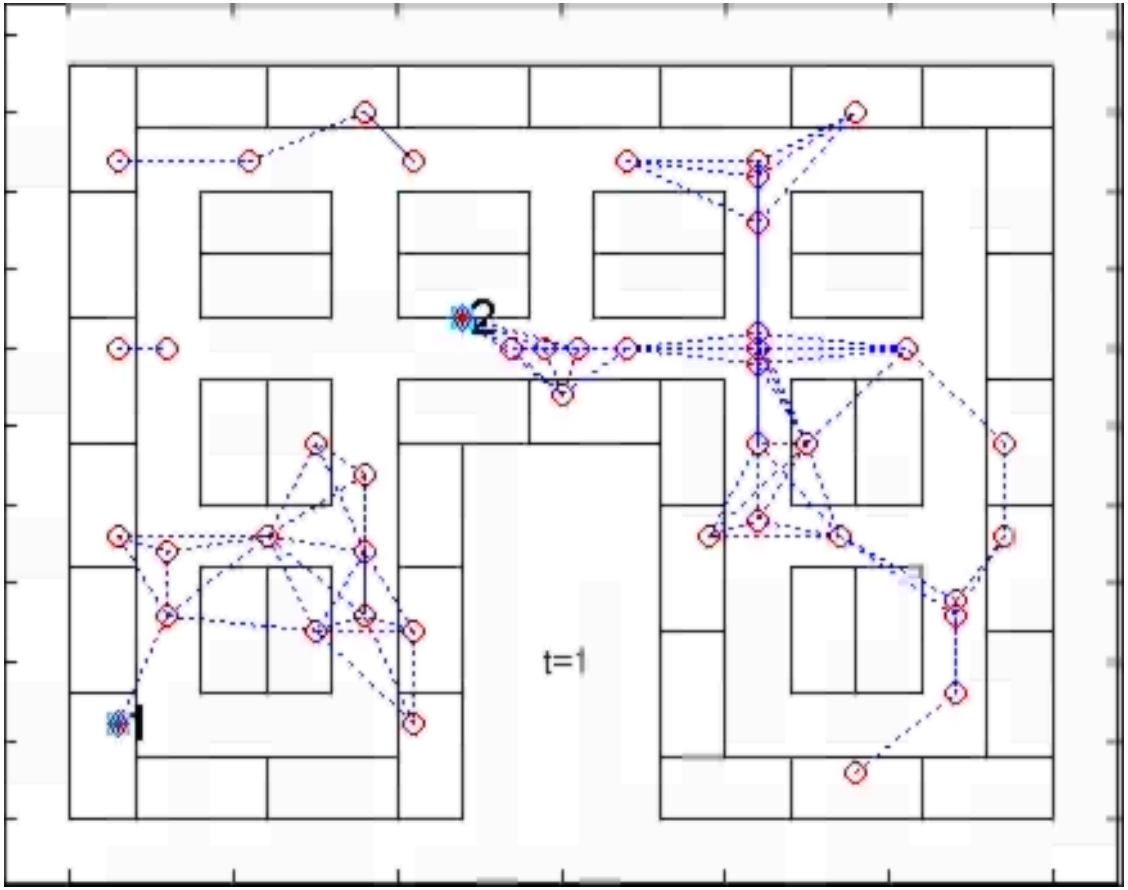
Weakly coupled oscillators

$b = 3,\ \varepsilon = 0.05$

Mobile Ad Hoc Networks



Mobile Ad Hoc Networks



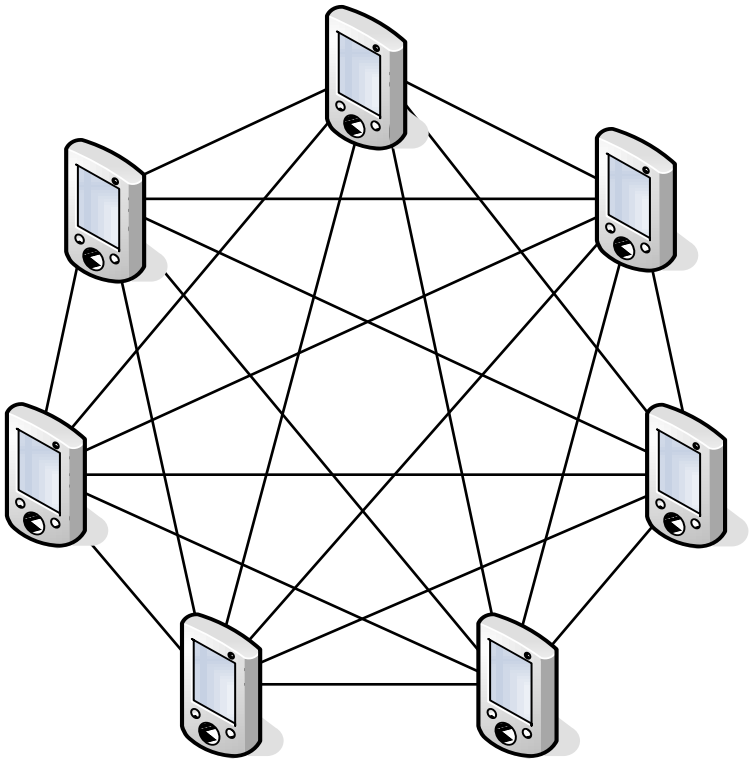
Why synchronization is important in mobile ad hoc networks

At physical layer

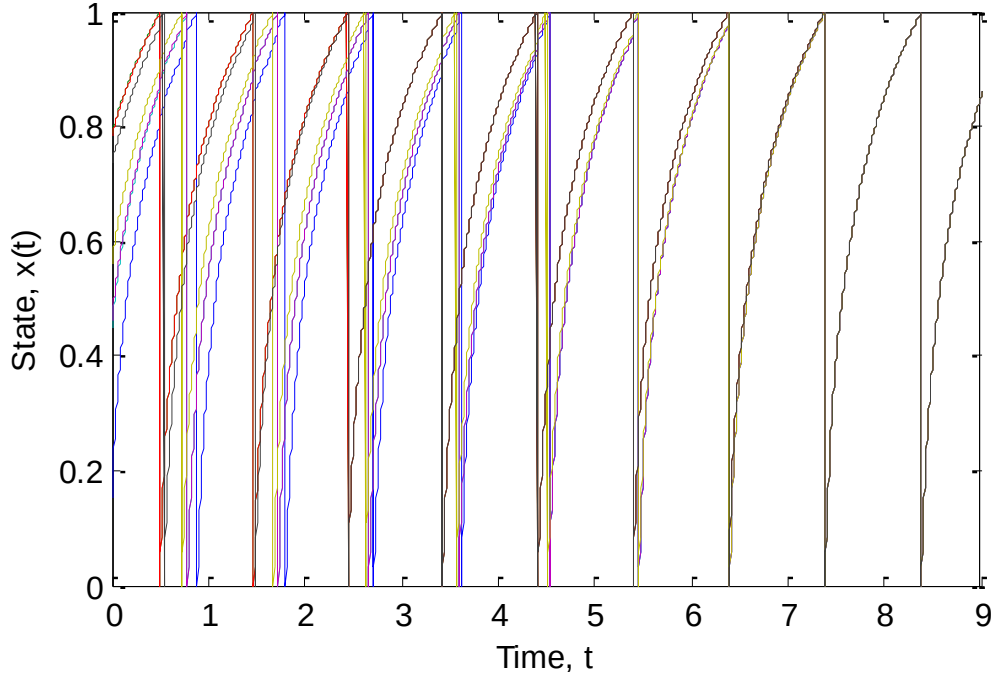
- Frequency Hopping

- **At multiple access layer**
  - Packet scheduling
- **At network layer**
  - Temporary assignment of transmission resources
- **At Transport Layer**
  - Synchronization of transmission patterns for maximum utilization of resources
- **At application layer**
  - Consistent ordering of events in sensor networks

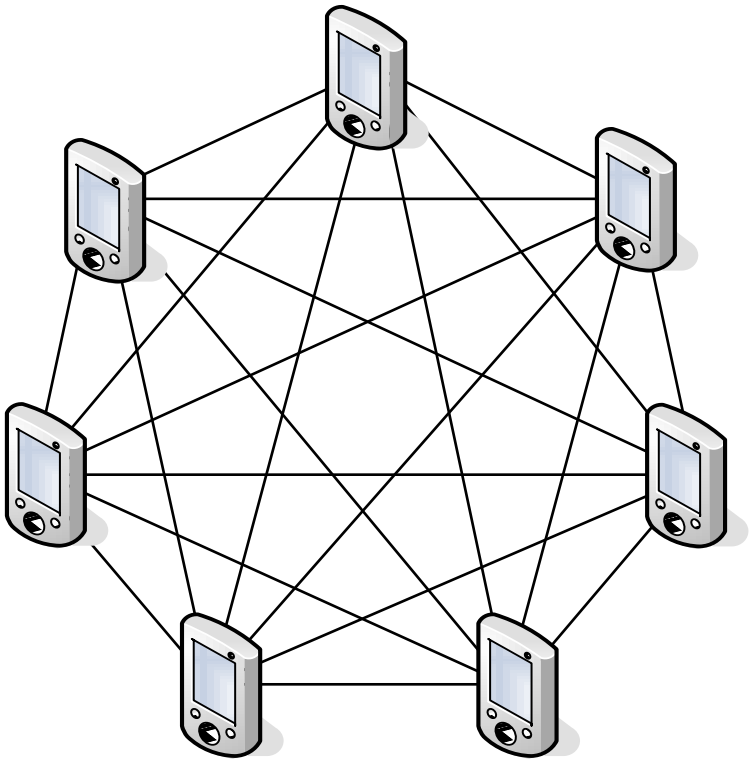
MANET Bio-inspired Synchronization

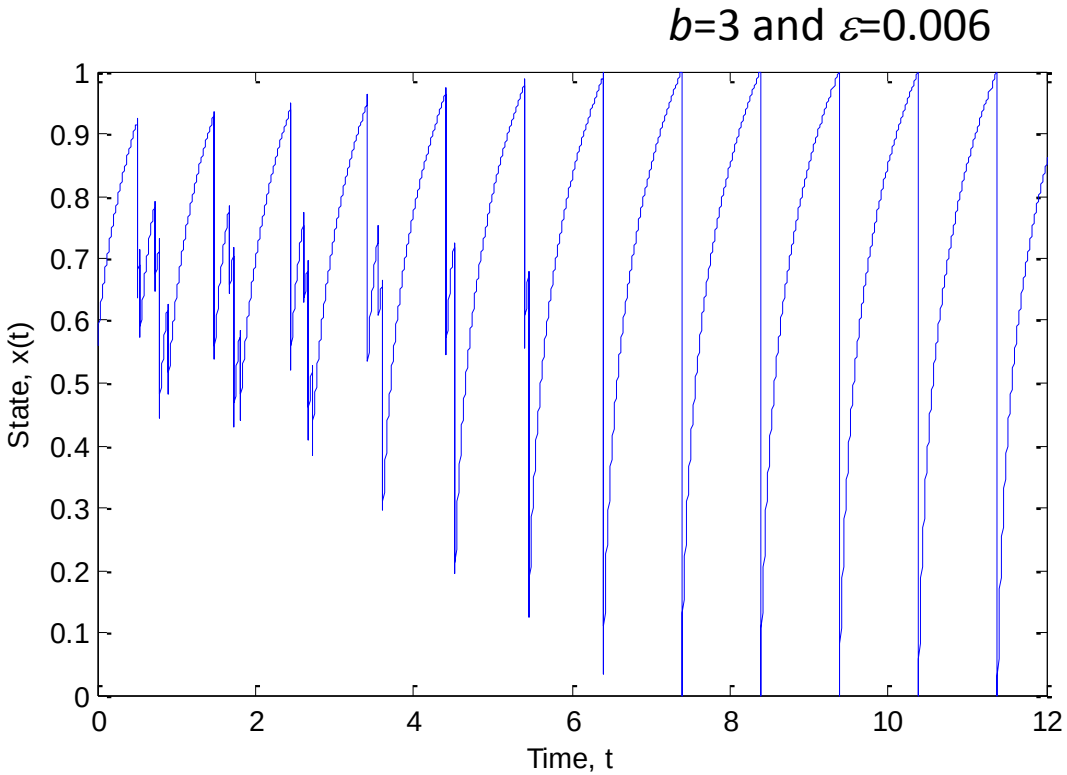


b=3 and ε=0.006



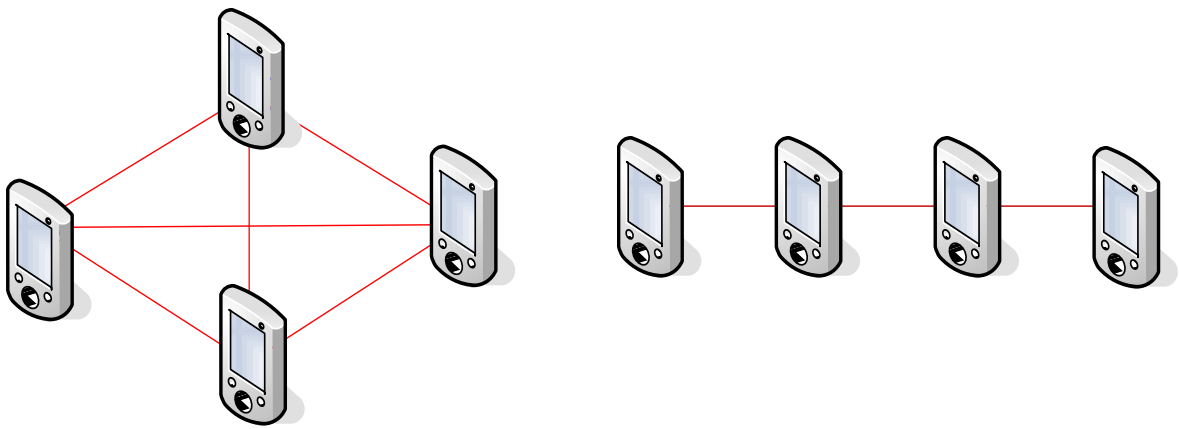
MANET Bio-inspired Synchronization





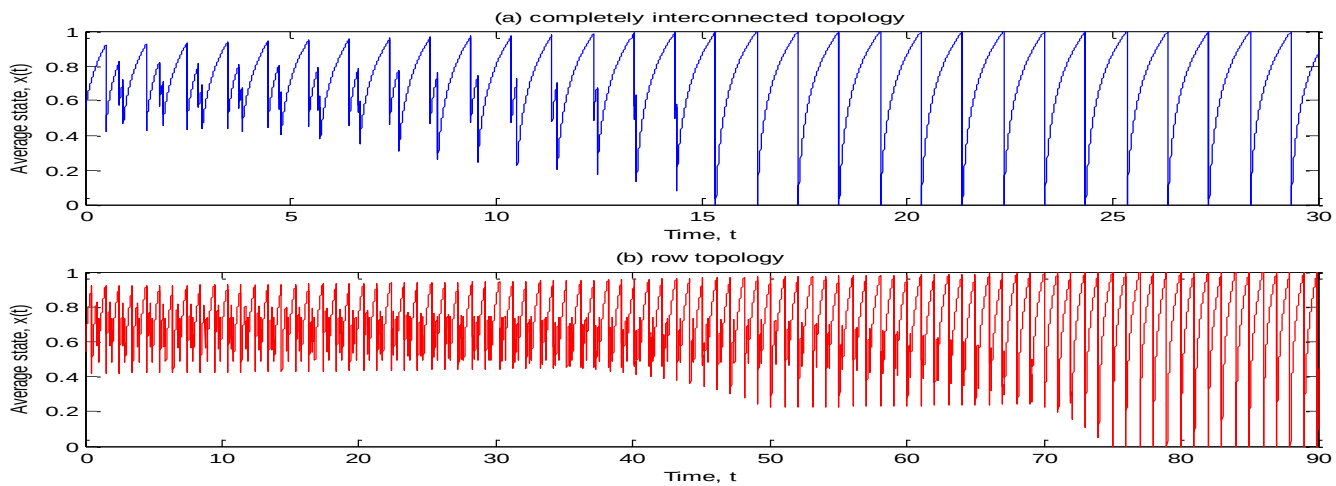
Average state among seven nodes, another view of synchronization

Effects Of topology



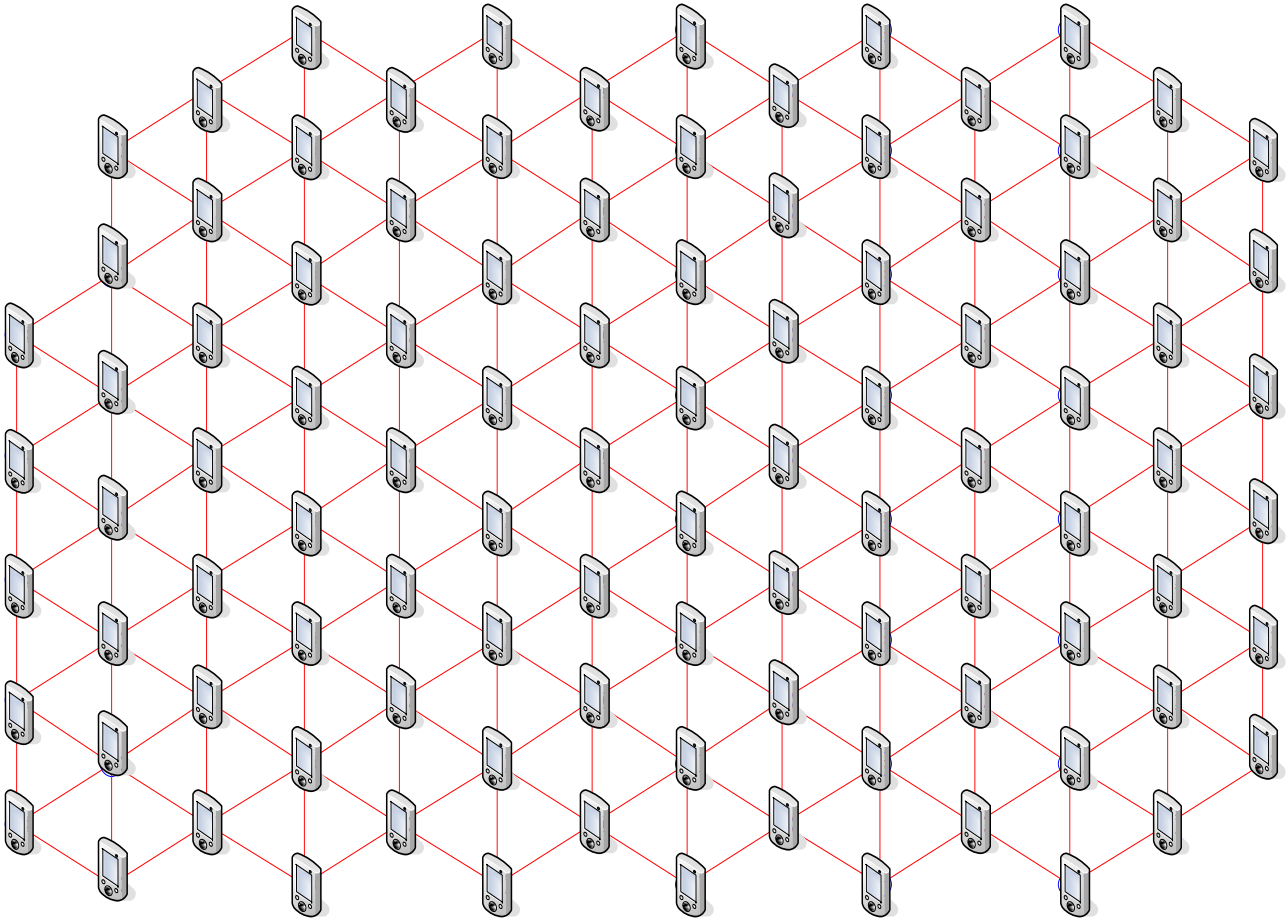
(a) Four nodes completely interconnected (b) Four nodes in a row

Two topologies in a four-node MANET



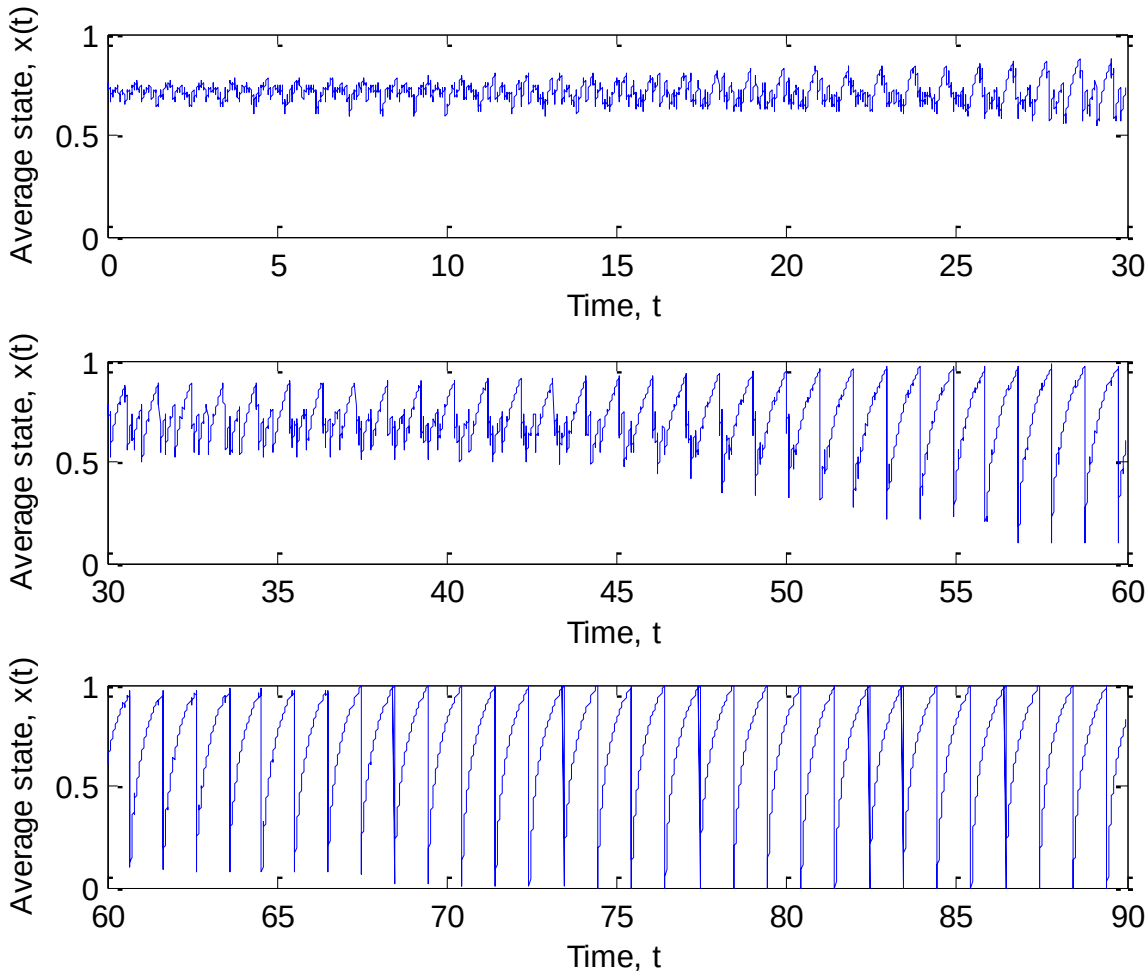
For Synchronization the first topology requires 17 pulses, the second topology requires 75.

Effects Of topology



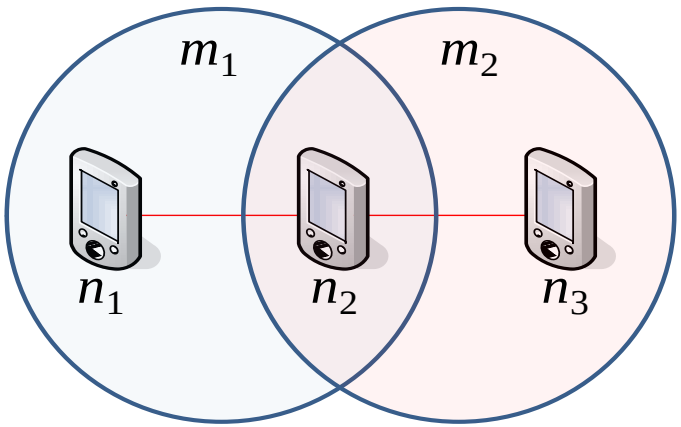
100 nodes in a regular topology

Effects of topology



Average state among the 100 nodes

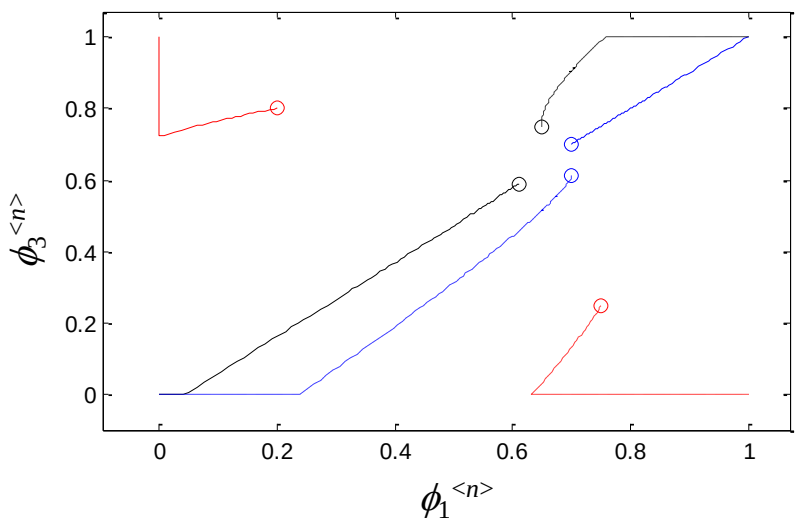
Effects of topology



$$(\phi_1, 0, \phi_3) \rightarrow (\phi_1', 0, \phi_3')$$

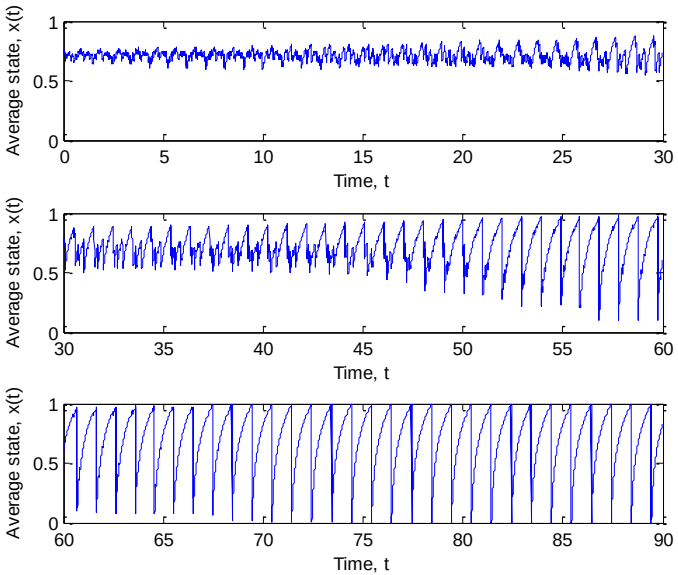
$$\phi_1' = g(\varepsilon+f[1+\phi_1-\phi_3-g(\varepsilon+f[\phi_1-\phi_3+g(\varepsilon+f[1-\phi_1])])])$$
$$\phi_3' = g(\varepsilon+f[1-g(\varepsilon+f[\phi_1-\phi_3+g(\varepsilon+f[1-\phi_1])])])$$

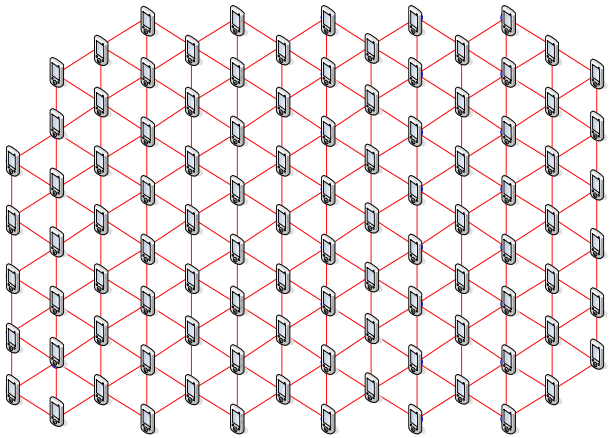
The only equilibrium point of this 4-dimensional return map is unstable: As long as there are no partitions, a network will get synchronized



All trajectories in the phase space lead to synchronization

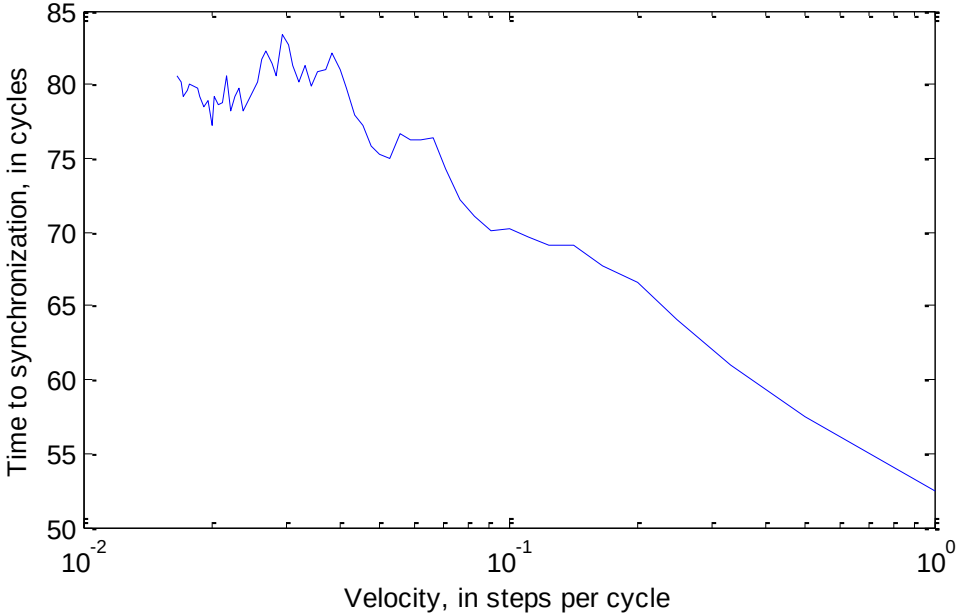
Effects of Mobility



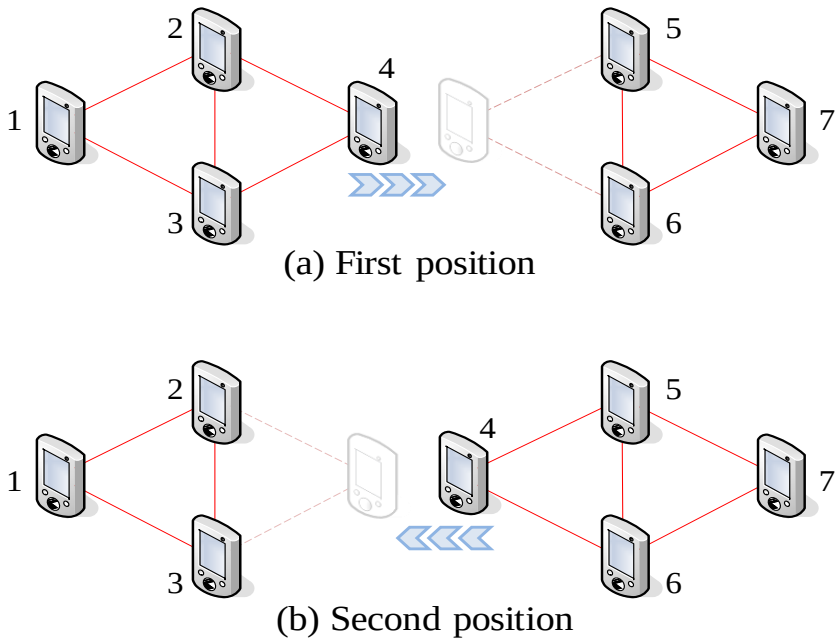


74 pulses for synchronization for non-moving nodes

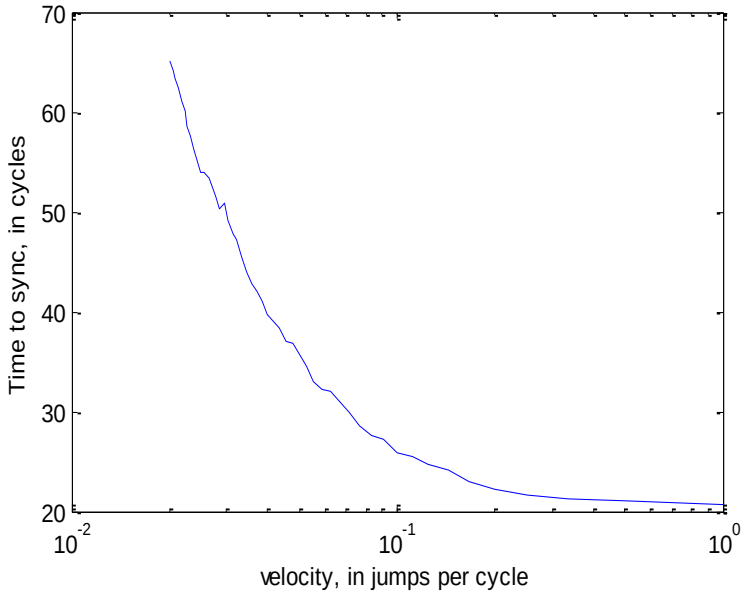
Mobility accelerates the dissemination of synch information



Effects of Mobility

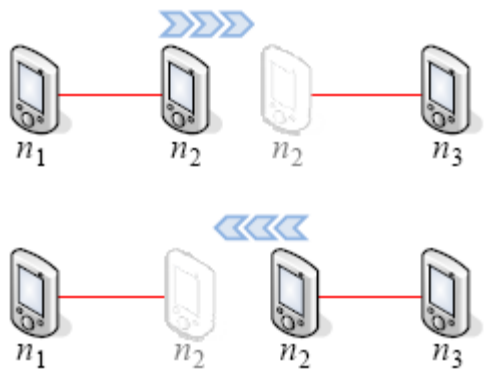


A Dynamic partitioned network

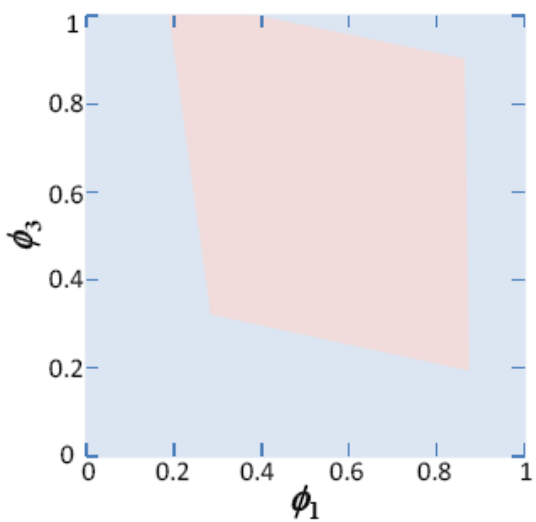


Through mobility, the clusters of a partitioned network get synchronized

Effects Of mobility



Three nodes interacting indirectly through mobility

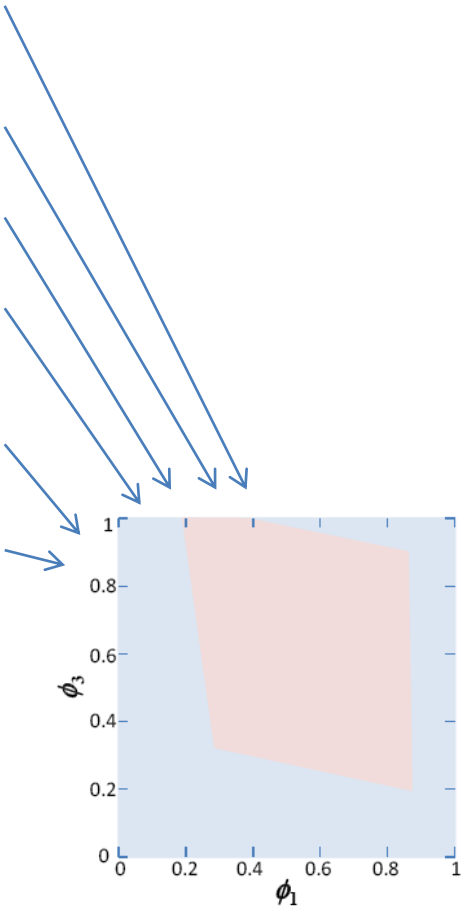


$$R\left(\begin{bmatrix}\phi_1\\ \phi_3\end{bmatrix}\right)=\begin{bmatrix} h\big(1-h(1-\phi_1)\big)+h(1-\phi_1)+\phi_1-\phi_3-h\big(h(1-\phi_1)+\phi_1-\phi_3\big) \\ h\big(1-h(\phi_1-\phi_3+h(1-\phi_1))\big)\end{bmatrix}$$

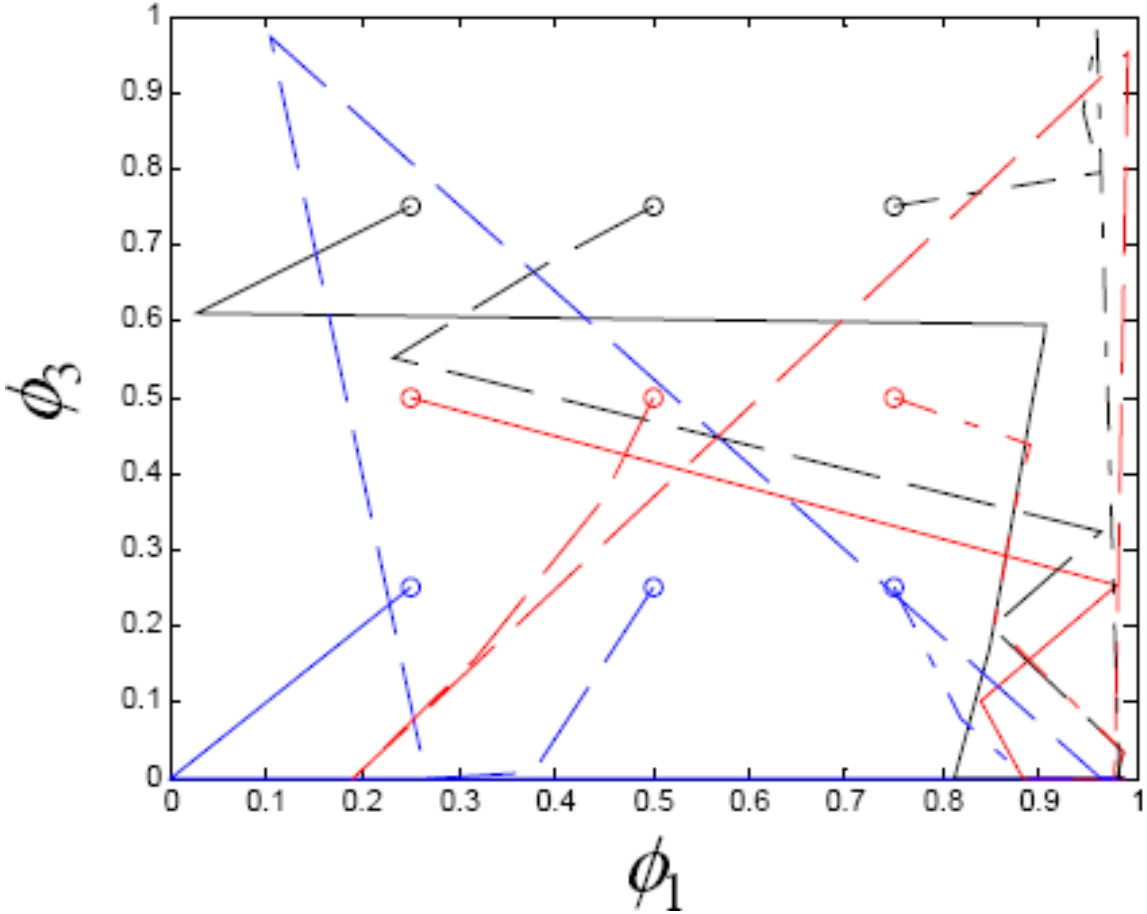
Region where the given return map holds, for *b=3 and ε=0.06,* where $h(\alpha) \equiv g(\varepsilon+f(\alpha))$

Effects of Mobility

Time	Phases	Event
0	$(\phi_1, 0, \phi_3)$	n_2 has just fired, leaving the given phases, and moves near n_1, which is the next node to fire
$1-\phi_1$	$(0, h(1-\phi_1), 1-(\phi_1-\phi_3))$	n_1 has just fired, affecting n_2. Next node to fire is n_3.
$1-\phi_3$	$(\phi_1-\phi_3, \phi_2, 0)$ where $\phi_2=h(1-\phi_1)+\phi_1-\phi_3$	n_3 has just fired, affecting none. Next node to fire is n_2.
$2-(\phi_2+\phi_3)$	$(\phi_1', 0, 1-\phi_2)$ where $\phi_1'=h(1-h(1-\phi_1))$	n_2 has just fired, affecting n_1, and moves near n_3. Next node to fire is n_1.
$3-(\phi_1'+\phi_2+\phi_3)$	$(0,1-\phi_1', 2-(\phi_1'+\phi_2))$	n_1 has just fired, affecting none. Next node to fire is n_3
$2-\phi_3$	$(\phi_1'+\phi_2-1, h(\phi_2), 0)$	n_3 has just fired, affecting n_2. Next node to fire is n_2.
$3-\phi_3-h(\phi_2)$	$(\phi_1'+\phi_2-h(\phi_2),0, h(1-h(\phi_2))$	n_2 has just fired, leaving the given phases, and moves near n_1. This is the given return map.



Effects of Mobility



Phase space trajectories of nodes n_1 and n_3, at firing instants of node n_2, immediately before moving to the first position

Conclusions

- Firefly synchronization in wireless ad hoc networks does not need to rely in a completely interconnected topology.
- Even under the case of partitioned networks, mobility can establish positive feedback interactions among the nodes, so synchronization can emerge easily in temporally partitioned mobile ad hoc networks.
- To accelerate the synchronization process, the interactions among nodes must be encouraged, either by forming dense clusters or by increasing mobility.

  - (By the way, This is an ongoing research work in which we want to keep synchronism within a MANET to coordinate bandwidth estimation procedures, transmission schedule, and resource reservation schemes).

Gracias! (Thank you ☺)

Questions?

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