Magnetic Suspension Control

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Summary

In this paper we present the design of a control system for a Magnetic Levitator. Simulated results are presented and analyzed for the open and closed loop systems.

1. Introduction

We consider a small steel ball of mass m placed below an electromagnet at a distance y. We design a control system to keep the ball suspended at its equilibrium point. The control law is the voltage applied to the electromagnet. A diagram of the system is shown in figure 1.

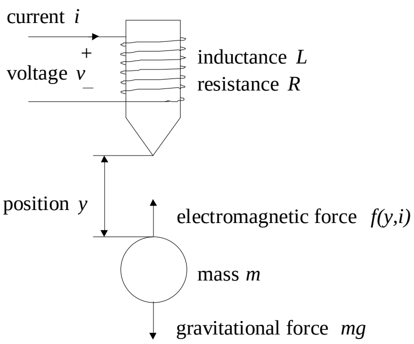

Figure 1. Diagram of the levitation system

1.1 Dynamic Equations

Newton's Law gives the equation of motion of the ball:

$$m \cdot \ddot{y} = -f(y,i) + m \cdot g \qquad (1)$$

The force $f(y,i)$ caused by the magnetic field is given by

$$f(y,i) = \frac{L_0}{2a} \cdot \frac{i^2}{(1 + y/a)^2} \qquad (2)$$

where a and L_0 are constants to be measured experimentally. Kirchoff's voltage law gives the voltage applied to the inductor:

$$v = R \cdot i + \tfrac{d}{dt}\left[L \cdot i\right] = R \cdot i + L \cdot \dot{i} \qquad (3)$$

Note that the inductance L is a function of the position y, so it must be included in the derivative. However, since our purpose is to keep the ball close to its equilibrium point, we assume L is constant.

We consider the following set of state variables: $x_1 = y$, $x_2 = y'$ and $x_3 = i$. So the above dynamic equations can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\dfrac{L_0}{2 \cdot m \cdot a} \cdot \dfrac{x_3^2}{(1 + x_1/a)^2} + g \\ \dfrac{v}{L} - \dfrac{R}{L} \cdot x_3 \end{bmatrix} \qquad (4)$$

1.2 Linearization

At its equilibrium point ($y=y_0$, $v=v_0$), the magnetic force $f(y_0,i_0)$ must be equal to the gravitational force mg, for which we must satisfy the following relationship

$$v_0 = \sqrt{\frac{2mga}{L_0}} \cdot R \cdot \left(1 + \frac{y_0}{a}\right) \qquad (5)$$

Taking the first terms of the Taylor expansion of (4) around this equilibrium point, and evaluating them at the equilibrium point, we get

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2 \cdot g}{a + y_0} & 0 & -\frac{\sqrt{2 \cdot L_0 \cdot m \cdot g \cdot a}}{m \cdot (a + y_0)} \\ 0 & 0 & -R/L \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix} \cdot v \qquad (6)$$

where (x_1, x_2, x_3) have been redefined to be the displacements of the previously defined state variables with respect to their equilibrium points and v has been redefined to be the displacement of the input voltage with respect to its equilibrium value (5).

For measuring the position we assume we have a light source and a photoreceiver system such that, around the equilibrium point y_0, the output voltage varies linearly with the ball position. So the output equations become

$$z = \begin{bmatrix} c_1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (7)$$

where c_1 is the gain of the position sensor. We could also measure the current, but it is easy to estimate, as we will see.

2. Analysis of the open-loop system

According to equations (6) and (7), let us define A, b and c as follows

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2 \cdot g}{a + y_0} & 0 & -\frac{\sqrt{2 \cdot L_0 \cdot m \cdot g \cdot a}}{m \cdot (a + y_0)} \\ 0 & 0 & -R/L \end{bmatrix}, \qquad b = \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix}, \qquad c = \begin{bmatrix} c_1 & 0 & 0 \end{bmatrix} \qquad (8)$$

The characteristic polynomial of the matrix A is

$$s^3 + \frac{R}{L} s^2 - \frac{2g}{a + y_0} s - \frac{2gR}{(a + y_0)L} \qquad (9)$$

which has three real roots

$$\lambda_1 = -\frac{R}{L} \qquad \lambda_2 = -\lambda_3 = \sqrt{\frac{2g}{a + y0}} \qquad (10)$$

Clearly, λ_2 is a positive eigenvalue of A, so this system is unstable. To stabilize it we need to move this eigenvalue to the left semiplane. This is possible because the system is controllable (in effect, as shown in (11) below, the controllability matrix is non-singular).

$$\left\|\begin{bmatrix} b & Ab & A^2b \end{bmatrix}\right\| = \left\|\begin{bmatrix} 0 & 0 & -\frac{\sqrt{2L_0mga}}{mL(a+y_0)} \\ 0 & -\frac{\sqrt{2L_0mga}}{mL(a+y_0)} & \frac{R\sqrt{2L_0mga}}{mL^2(a+y_0)} \\ \frac{1}{L} & -\frac{R}{L^2} & \frac{R^2}{L^3} \end{bmatrix}\right\| = -\frac{2L_0ga}{mL^3(a+y_0)^2} \neq 0 \quad (11)$$

Testing for observability, we see that measuring the ball position is enough to know the state trajectory :

$$\left\|\begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}\right\| = \left\|\begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_1 & 0 \\ \frac{2\cdot c_1\cdot g}{a+y_0} & 0 & -\frac{c_1\sqrt{2\cdot L_0\cdot m\cdot g\cdot a}}{m\cdot(a+y_0)} \end{bmatrix}\right\| = -\frac{c_1^3\sqrt{2\cdot L_0\cdot m\cdot g\cdot a}}{m\cdot(a+y_0)} \neq 0 \qquad (12)$$

In conclusion, we have a controllable, observable and unstable system. For simulation purposes we consider the following parameter values:

m = 20 g,	R = 20 Ω,	L = 500 mH,	L_0 = 25 mH,	(13)
a = 7 mm,	g = 9.8 m/s^2,	y_0 = 4 mm,	c_1 = 3 V/mm	.

for which the equilibrium point is achieved with v_0 = 10.4 V and the linearized state-space model becomes

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1781.8 & 0 & -37.65 \\ 0 & 0 & -40 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \cdot v \qquad (14)$$

$$z = \begin{bmatrix} 3000 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad .$$

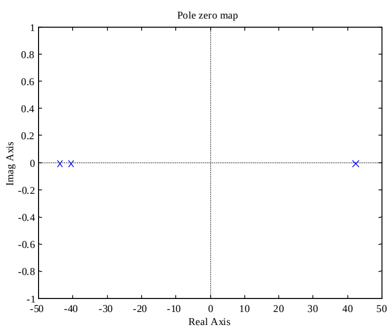

Figure 2. Pole-zero map

The eigenvalues of A are –40 and ±42.2 (figure 2). To observe how the positive eigenvalue affects the stability, in figure 3 we plot the step, impulse and frequency responses of the system.

A small increment in the input voltage produces a corresponding increment in the coil current. This will increment the electromagnetic force attracting the ball to the magnet. But this decrease in the distance will increase again the electromagnetic force in a positive feedback that finally sends the ball upwards until it is captured by the magnet after a few milliseconds.

The first thing you think about to solve this problem is to use output feedback to stabilize the system. However, the root locus shown in figure 4 indicates that there is no a gain k such that $v(t) = -k\, y(t)$ stabilize the system. In effect, the positive pole moves to the right, augmenting the unstability.

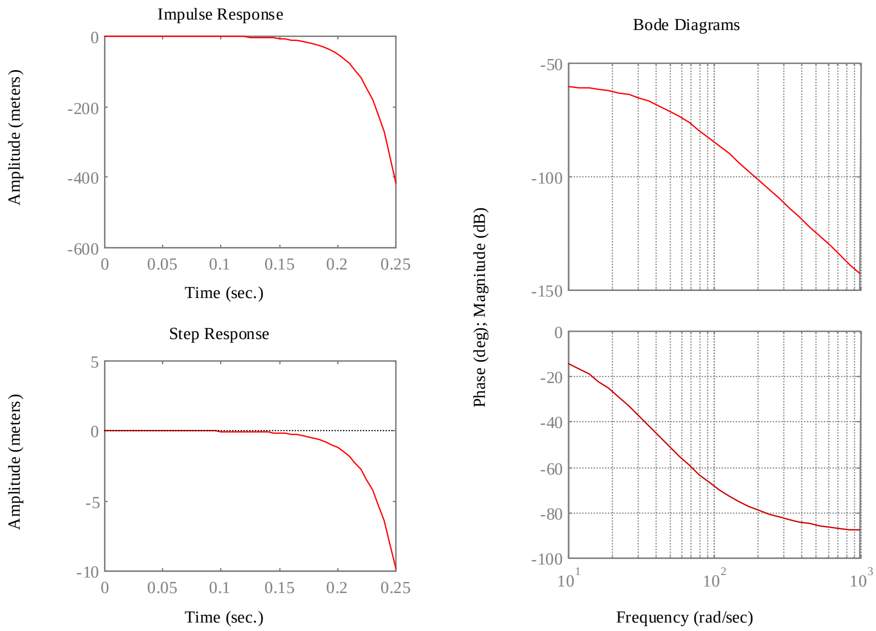

Figure 3. Time and frequency responses of the open loop system

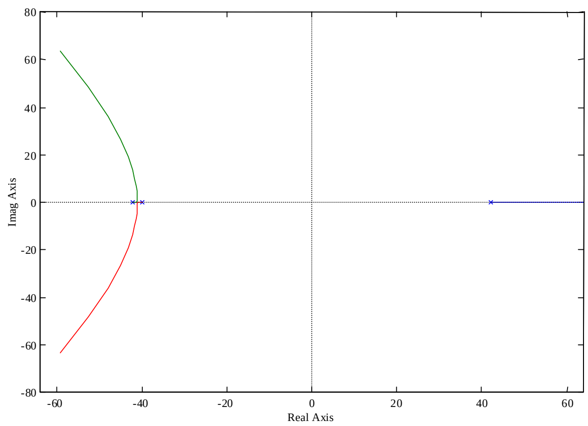

Figure 4. Root locus for output feedback

We do not use dynamic compensation to move the root locus because there is no a compensator that allows us to obtain a good response. For example, figure 4(a) shows the root locus with a compensator *(s+20)/(s+60)*. For this particular case we can stabilize the system using any feedback gain between -2840 and -2900, but the real part of at least one of the roots, though negative, is greater than -1.

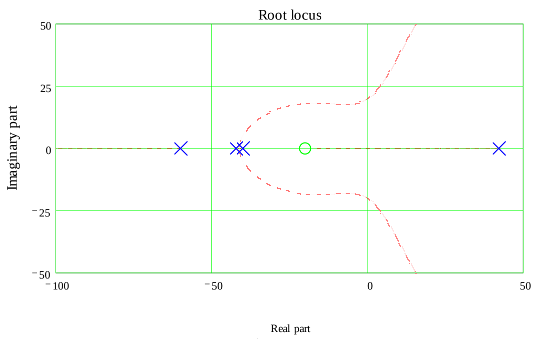

Figure 4(a). Root locus for the compensated closed loop system

3. State feedback

Since the system is controllable, we can locate its poles wherever we can by using the appropriate control law as a linear combination of the state variables. After testing several alternatives, we choose to put the poles at -40 and -50 ± 50*j*. In effect, as figure 5 shows, we obtain a nice response and the feedback gains are easily attainable with actual sensors and actuators: 6 V/mm, 0.143 V/(mm/s) and 50 Ω.

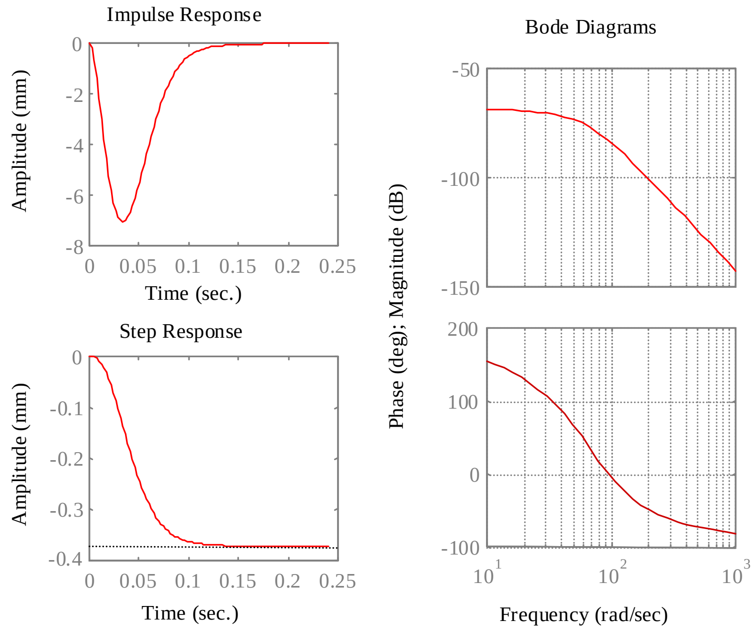

Figure 5. Time and frequency responses of the closed loop system

4. Observer Design

The above nice design cannot be implemented as it is, simply because we do not have access to all the state variables. By means of a photodetector, we are measuring the position of the ball. Maybe we can also measure the current in the coil, but definitively it is not a good idea trying to devise a sensor for the velocity of the ball. But since the system is completely observable just by measuring the position of the ball, we decided to use it to estimate both the current in the coil and the velocity of the ball.

It would be desirable to locate the poles of the observer 10 times farther than the closed loop poles. But just to avoid too high gains in the feedback path, we located the observer poles at -200 and -250 $\pm 250j$. Figure 6 shows how the estimated position tracks the actual position until it gets a perfect match in a very short time compared with the response time of the closed loop system.

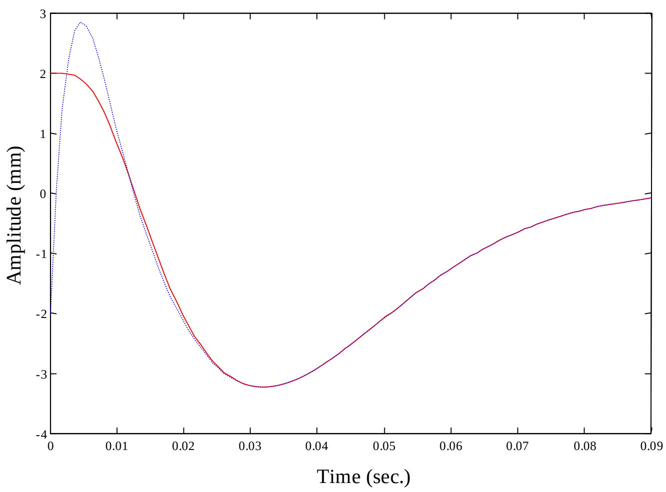

Figure 6. Actual (red) and estimated (blue) position of the ball

The observer we have just designed adds three more state variables to the system, for a total dimensionality of six. But we can reduce this dimension to five if we take into account that, being measuring the position of the ball, we do not need to estimate it. So we also designed a reduced order observer for the velocity of the ball and the current in the coil. The realization of the whole system has dimension five.

Now we use the feedback gain obtained in numeral 3 above to feedback the estimated current and velocity as well as the actual position. The response obtained is exactly the one shown in figure 5.

5. Linear Quadratic Regulator

Instead of locating the closed loop poles arbitrarily, we can choose the feedback gain to minimize some cost function. We want the ball to be as close as possible to its equilibrium position but we also want the input voltage to be as close as possible to its nominal value (any variation will imply an additional a.c. superimposed signal, which represents more energy dissipated in the control). Considering that one-millimeter of deviation in the position of the ball is as costly as one volt of deviation in the input voltage, the function to be minimized becomes

$$J = \int_0^{\infty} 1 \cdot 10^6 \cdot y(t)^2 + v(t)^2 \, dt \qquad (15)$$

The optimal feedback gain becomes 4.5 V/mm, 0.1 V/(mm/s) and 45.5 Ω, which is less than the feedback gain we designed before. The result of that decrease in the feedback gain is that the poles of the closed loop system have been moved to -59.2 and -35.9 ± 21.5*j*, closer to the right half plane and to the real axis, so the system has been slowed down a little. This means that our original design was a little bit too expensive in terms of consumed energy.

Figure 7 compares the impulse response of three of the systems we have designed here: (1) moving the unstable pole to the left half plane, (2) locating the closed loop poles at -40 and -50 ± 50*j*, and (3) optimizing the cost represented in equation (15).

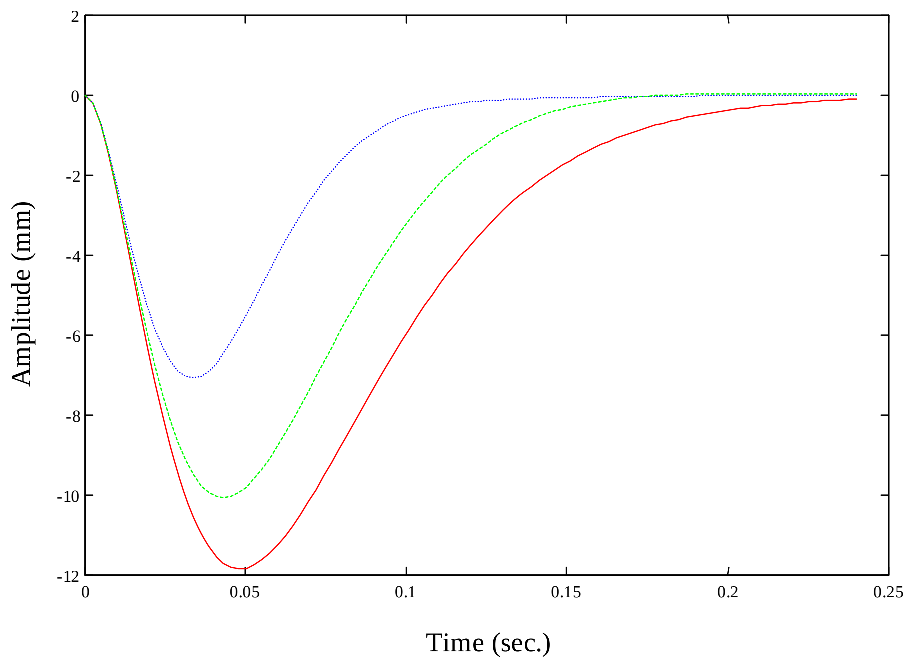

Figure 7. Impulse response of systems (1) -red-, (2) -blue- and (3) -green-.

6. Robustness

Keeping the LQR feedback gain and the reduced observer gain obtained for the parameter values of equation (13), we simulate the behavior of the system for different balls. This is a small and empirical test for robustness, but it gives some confidence on the system. In effect, although it was designed for 20-gram balls, it can suspend in the air balls three times heavier and hundreds of times lighter than the nominal one. Figure 8 shows the impulse response for balls of 5 grams, 20 grams and 50 grams.

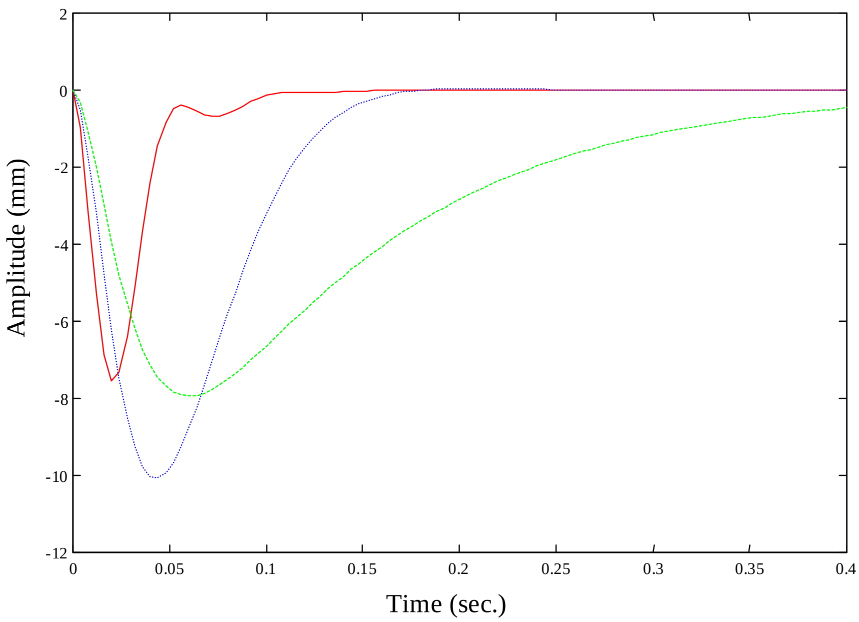

Figure 8. Impulse response for balls of 5 grams (red), 20 grams (blue) and 50 grams (green)

7. Conclusions

We have designed several control systems for suspending a steel ball in the air. In the process we have used many of the concepts and techniques that must be learnt during an introductory control course: (1) state, input and output variables recognition, (2) quantitative formulation of control objectives, (3) model linearization, (4) stability, controllability and observability, (5) output feedback and root locus, (6) state feedback and pole assignment, (7) observers and reduced order observers, and (8) linear quadratic regulators.

We have also used some of the extensive capabilities of Matlab for analysis and design of control systems. In the appendix we have included a Matlab program listing describing the whole procedure.

References

1. Franklin, Powell and Emami-Naeini, "Feedback Control of Dynamic systems" 3rd edition, Addison Wesley, 1995. (In particular: example 2.21 (pg. 63), exercise 5.34 (pg. 327) and exercise 8.16 (pg. 656)).
2. V.A. Oliveira, E.F. Costa and J.B. Vargas, "Digital implementation of a magnetic suspension control systems for laboratory experiments", IEEE Transactions on Education, Vol. 42, No. 4, November 1999. (Equation (2) and practical values like those of equation (13) were taken from this paper).