HOMEWORK # 4

1. The given files include 12 textures (from `textura01.bmp` to `textura12.bmp`) and 12 natural objects (3 flowers (or flower seeds), 3 clouds, 3 wood textures and 3 iris patterns, from `prueba01.bmp` to `prueba12.bmp`), as shown in Fig. 2. Compute the box-counting dimension of each image and discuss whether this parameter is a good discriminator to tell woods from clouds from flowers from irises. Could you classify some of the original textures within your four classes?
   Note 1. So far, we have used the box-counting dimension algorithm to compute the dimension of a rough line in a plane. But in this homework we have to compute the dimension of a rough surface in a 3D volume (the grey level gives the third Euclidean dimension), so we have to count 3D boxes, not 2D squares[*].
   Note 2. If you have a set of geometrical 3D surfaces of your own in which you are interested because of your research project, please feel free to use it. All you have to do is to compute the box-counting dimension and decide if it is a good discriminator for different classes within your set (but using a surface in a 3D space instead of a line in a 2D space).

2. Compute the box-counting dimension of (a) a finite set of isolated points in the unit interval, (b) the unit interval in the real line and (c) the unit square in the plane.

3. What is the self-similar dimension of the set that results from taking out from the closed unit interval all the open sub-intervals whose decimal expansion includes the digit 5? (in the expansion, a 5 followed by an infinite number of 0's is changed by a 4 followed by an infinite number of 9's)

4. Starting with the unit interval, $I_0=[0,1]$, we define the following IFS
$$I_{n+1} = \{x/2: x\in I_n\}\cup\{(3+2x)/5: x\in I_n\}$$
   What is the Hausdorff dimension of the set of points that is left after an infinite number of iterations?

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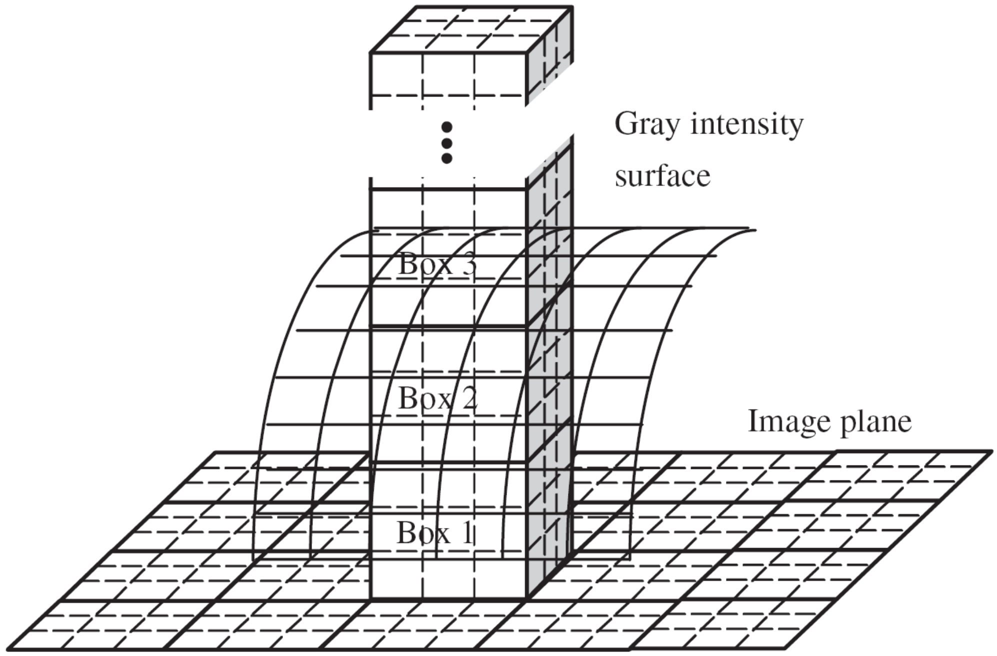
[*] Last class, we used the BoxCounting algorithm to compute the dimension of a rough line in a plane. But if we consider a grey level 2D image as a 2D surface in a Euclidean 3D space, where the grey level gives the third Euclidean dimension, we could use the same method to count the boxes. The following text is taken from [Jian Lia, Qian Dub and Caixin Suna. "An improved box-counting method for image fractal dimension estimation" Pattern Recognition 42(2009) 2460-2469]:

"Consider an image of size $M\times M$ as a 3D spatial surface with (x,y) denoting pixel position on the image plane, and the third coordinate (z) denoting pixel gray level. In the DBC method, the (x,y) plane is partitioned into non-overlapping blocks of size $s\times s$, where $1 < s \le M/2$ and s is an integer. N_s is counted in the DBC method using the following procedure. On each block there is a column of boxes of size $s\times s\times s'$, where s' is the height of each box, $G/s' = M/s$, and G is the total number of gray levels. For example, $s = s' = 3$ in Fig. 1. Assign numbers 1,2,...,to the boxes as shown in Fig. 1. Let the minimum and maximum gray level in the $(i, j)^{th}$ block fall into the k^{th} and l^{th} boxes, respectively. The boxes covering this block are counted in the number as
$$n_s(i, j) = l - k + 1$$
where the subscript s denotes the result using the scale s. For example, $n_s(i, j) = 3 - 1 + 1$ as illustrated in Fig. 1. Considering contributions from all blocks, N_s is counted for different values of s as
$$N_s = \sum_{i,j} n_s(i, j)$$
Then the BCD can be estimated from the least squares linear fit of $\log(N_s)$ versus $\log(1/s)$".


Gray intensity surface
Box 3
Box 2
Image plane
Box 1