Cognitive Cellular Automata for Image Segmentation: A Social Learning Metaphor

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ABSTRACT

Cognitive agents have the ability to perceive their environment and act on it according to models of reality built through memory, intelligence and language. Interacting cognitive agents interchange information about their models in order to build a collective knowledge of their reality (social learning). In this paper we use this distributed cognitive system paradigm to solve a segmentation problem in image processing from the complex systems engineering approach. We build a cognitive cellular automata where each pixel in the image is a cognitive agent. Social learning is achieved by stigmergic and direct communication among agents. Our results outperform typical segmentation methodologies for granular material. Our social cognitive learning metaphor exemplifies a complex systems engineering approach for more general applications.

Keywords: social learning, image processing, cellular automata, granular material, political formation, nature/nurture.

1. INTRODUCTION

In recent decades, there has been an enriching feedback between Science and Engineering. On the one hand, engineering design processes have been inspired by biological, physical, psychological, ecological and sociological phenomena (among many others) to solve computationally difficult problems [1]. On the other hand, engineering experimentation with these inspired processes has shed some light on the explanation of the corresponding scientific phenomena [2]. Here we explore a potential similar synergistic relationship between neuropsychology, sociology and complex systems engineering, related to the design of distributed cognitive dynamic systems.

Psychology and neurology study the cognitive abilities of human beings, that is, the ability of man to perceive his environment and act on it according to mental models that arise from perception/action cycles mediated by memory, attention, intelligence and language [3]. Sociology studies the collective phenomena produced by human beings, that is, the behavior of human beings when they are living together in a shared habitat, giving rise to culture and history [4]. The complexity sciences study systems that exhibit emergency and selforganization, that is, systems composed of many parts that, when interacting, give rise to new macroscopic qualities of the system in the form of spontaneous structures, more as an effect of the interactions than an effect of the functional contribution of the parts [5]. There is no doubt, then, that neuropsychology, sociology and the engineering of complex systems have many pending dialogues.

Sociological phenomena arise from the interaction of human beings. Arguably, these phenomena are emergent characteristics produced by self-organization through cognitive interactions [6]. This idea can inspire engineering procedures for complex systems. Our purpose in this paper is to present a basic example of a distributed dynamic cognitive systems design, and to discuss it in the context of neuropsychology and sociology, as a social learning process among cognitive agents.

The engineering problem in question corresponds to the segmentation of a noisy pavement image into two classes of regions, asphalt and gravel (Figure 1) [7]. The social metaphor is the formation of liberal and conservative opinions as a society evolves from childhood to maturity. Each pixel is classified as gravel or asphalt like each individual in the society becomes liberal or conservative. Each pixel begins with an initial gray level, so that dark pixels are likely to become asphalt and light pixels are likely to become gravel. Similarly, there could be natural genetic conditions that make children likely to become liberals or conservatives. However, the local environment also determines how some dark pixels could belong to gravel areas and light pixels to asphalt regions, as family values, education and friends help to determine political tendencies of a teenager. Finally, a careful model based classification should help to refine the final decision of whether a pixel should be classified as gravel or asphalt, as mature citizens use more rational criteria and discussions with fellow citizens to decide a particular behavior



Figure 1. (a) Original pavement image. (b) Asphalt and gravel segmentation

This paper use the metaphor of political opinion formation to classify pixels in pavement images. Initially, the pixels are dark or light indicating some "childhood tendencies", although they are not very clear about their positions. Then, "teenager pixels" tend to get closer to similar pixels and to take distance from different pixels just by imitation of their closer friends, without too much communications nor rationalization. Finally, "mature pixels" build a model of their environment and interchange information among neighbor pixels in order to make a rational decision of its own classification as asphalt or gravel. We believe this process is close to that of liberal or conservative opinion formation in a society as its members grow nurturing from nature tendencies.

The paper is organized as follows. After this brief introduction, a first social learning approach to segmentation is presented in section 2. This approach leads to the cognitive cellular automata model presented in section 3. This model works in two phases: a "teenager"

cellular automata in which each pixel tries to get closer to similar neighbor pixels and move away from different neighbor pixels (as presented in section 4), and a "mature" cellular automata where pixels interchange knowledge to decide their own class (as presented in section 5). Section 6 presents some results and section 7 conclude the paper.

2. SOCIAL COGNITIVE APPROACH

The gravel/asphalt image segmentation is not an easy problem. We built a simple drawing program to allow a human being to segment an image manually, so several graduate students tried to do it. The results are uncertain in the sense that different students obtained very different segmentations for the same image, and even the same student obtained different segmentations at different times for the same image. Anyway, this manual process, depicted in Figure 2, gives better results than common automatic segmentation procedures.



Figure 2. Manual segmentation of the images

We consider the graduate student is a cognitive system: She perceives the color image, I_C , and uses all her relevant knowledge acquired during her undergraduate studies in engineering and all her previous experience with similar problems in order to obtain a binary image, I_B , as a transformation of the original color image, $I_B=T(I_C)$. This mental binary model reflects her interpretation of the original color image, generating a sequence of actions to capture the mental image on the computer screeen through the drawing program. This perception/action cycle, mediated by a mental model of reality, is what we call the cognitive process. The notorious differences among segmentations of the same image by different students show that the mental process through which action is decided from perception through intelligence, memory and attention can be very different from one student to another, although all of them are good segmentations (Figure 3).



Figure 3. Different students produce different segmentations

Neuropsychology have taught us a lot about this internal decision process that controls the perception-action cycle [8]. The great neuroscientist J.Fuster considers it as a flow of information between the cognitive agent and its environment, from sensory organs to motor effectors that change the environment, which leads to new perceptions

and new actions, until some goal is achieved [9]. Dr. Fuster finds that the perception/action cycle is physically represented by "cognits" (subnetworks of cortical neurons that are constructed during Hebbian forming learning, а hierarchical structure of perception/memory/action) [10]. This fundamental idea has inspired many engineers, as formalized in the work of Dr. Simon Haykin [11], to build the Theory of Dynamic Cognitive Systems. It uses statistical signal processing, stochastic control, information theory, statistical learning and game theory to simulate the cognitive abilities of the human being (perception, memory, attention, intelligence, language, action), according to neuroscience.

The fact that several students obtain very different segmentations for the same color image can be attributed to different reasons. For example, at the most basic perception, some students have better vision than others. More psychologically, different aspects of the image can be interesting to focus attention, so the student concentrates on some visual stimuli while ignores others (e.g., those more interested in plastic arts are more careful delineating the gravel). In the action part of the cycle, some students have better fine motor skill than others. The mood of the students also has a great influence. However, if they work on the images segmented by other students, the results tend to converge on a much more satisfying segmented image. The process is depicted in Figure 4. This leads us to the second theme of our approach: social learning.



Figure 4. Social interaction for better segmentation

First, student 1 applies her own transformation T_1 to the original image I_C and obtains the first segmented image, I_{B1} . This segmented image is given to student 2, along with the original image I_C , in order for her to apply her own transformation T_2 and obtain the second estimation of the segmented image, I_{B2} . This process is repeated with several students, each generating a new segmented image form the original image and the segmented image of the previous student. At each iteration, the students not only add minute details, but also shrink or stretch some gravel, joint different gravels in a bigger one, separate a single gravel into two smaller ones, etc. Many times, they reverse the changes made on previous iterations by other students. Although there is not a final consensus on which is the correct segmentation, each iteration leads to a more acceptable segmentation, so students get more satisfied each time, as shown in the sequence of iterations of Figure 5.



Figure 5. Segmentation refinement through social learning

Social psychology have taught us a lot about this social learning process that controls the distributed consensus among rational agents [12]. Recently, this kind of processes have attracted research interest in engineering for distributed estimation in sensor networks, consensus algorithms in robotic networks, distributed machine learning, synchronization in mobile ad hoc networks, cooperation emergence in cognitive radio networks, etc. [13] In particular, the topic of opinion dynamics over social networks, where agents can communicate only with a local group of agents, but beliefs are propagated through the network, is gaining more importance for its potential effects (good or bad) on democracy [14]. In our case, each student weights its own opinion on the correct segmentation with that of the previous student, which brings a summary of the opinions of all previous students. Presumably, the n^{th} student builds the following estimation:

$$I_{Bn} = T_n(I_C; I_{Bn-1}) = I_{Bn-1} + A_n \cdot (T_n(I_C) - I_{Bn-1})$$

where A_n is a matrix of 0's and 1's, the same size as the image, choosing for each pixel her own independent classification, $T_n(I_C)$, or that of the previous estimation, I_{Bn-1} . Consensus is achieved if A_n tends to a zero matrix as *n* increases [15]. In our case, most students reach a zero matrix after a few iterations, but some of them keep making very small changes in conflictive regions of the image (although most entries of their matrices reaches the value zero).

In this paper we explore how social learning among distributed cognitive agents can lead to an acceptable solution to the difficult problem of gravel/asphalt segmentation in pavement images. Instead of human cognitive agents perceiving the image and acting on it, we consider each pixel in the image as a cell in a cellular automata that becomes the cognitive agent in the distributed learning system.

3. THE COGNITIVE AGENT AND ITS ENVIRONMENT

The agent is the pixel, a cell in a cellular automata, which can perceive and act on a small neighborhood of the image, an array of 31×31 pixels around it. Each element of the array (each pixel) can take a particular gray level value between 0 and 255. A pixel with a zero value is black, a pixel with a value 255 is white and other values correspond to intermediate gray levels between black and white. The cell agent compute a bit according to its 31×31 neighborhood, so that a new I_B image is built, in the same domain of I_G but with co-domain $\mathbb{Z}_2 = \{0,$ 1}, where the pixels in zero state correspond to asphalt and the pixels in state 1 correspond to gravel.

The agents have simple perception/action capabilities: They can perceive the gray value of each pixel in its 31×31 neighborhood and they can change their own gray value.

In order to decide what action to take, we consider the whole image as a society that evolves with time according to local social interactions among the members. A pixel being asphalt o gravel is like a person being liberal or conservative. In a first stage, the agents are not very clear about their positions, but they like to get closer to similar agents and to take distance from different agents, without too much communications nor rationalization, just by imitation of their closer friends. This way, they form a segregated society of teenagers with many tribes in a continuum of political positions, which are easy to discretize in a number of groups with some liberal or conservative tendencies.

In a second stage the agents get more rational (more mature?) and polarize the society into two types of pixels according to a more elaborated procedure: They communicate among them and evaluate the maximum-a-posteriori probability of being part of one group or another according to their perceptions, the communication with neighbor agents, their experience as teenagers, and their own believes. For this second stage, those agent that did make up their minds early in time become references for other doubtful agents in the rational stage.

4. A SOCIETY OF TEENAGERS

In the first stage, each agent considers the range of gray values within its neighborhood. If all of them are very similar, the agents moves to the average, wanting to belong. If there are significant differences, however, they timidly moves toward those a little bit closer to them. As this process is repeated in a cellular automata of local interactions, different tribes are formed. The algorithm is as follows.

Given the gray image in the neighborhood of an agent, I_G , it can compute the maximum, the minimum, the average and the range of the gray levels:

$$mx = \max_{(x,y)} I_G(x, y)$$
$$mn = \min_{(x,y)} I_G(x, y)$$
$$\mu = \frac{1}{N} \sum_{(x,y)} I_G(x, y)$$
$$r = (mx - mn) / mx$$

where (x,y) runs over all pixels in its neighborhood. The quantity r talks of the homogeneity (low r) or heterogeneity (high r) of the ideas in the neighborhood. If it is low, the agent wants to belong and changes its gray level closer to the mean:

$$I_G(x) \leftarrow r \cdot I_G(x) + (1-r) \cdot \mu$$

The lower r, the closer the agent gets to the mean in a single step. If r is high, there are different ideas in the neighborhood and the agents gets closer to the people that enforce its own ideas:

$$I_G(x) \leftarrow \begin{cases} r \cdot mn + (1-r) \cdot I_G(x) & \text{if } I_G(x) < \mu \\ r \cdot mx + (1-r) \cdot I_G(x) & \text{if } I_G(x) \ge \mu \end{cases}$$

The higher r, the closer the agent gets to the corresponding local extreme in a single step. Figure 6 shows a sequence of interactions of this cellular automata in a profile of pixels. Sharp transitions are emphasized and small transitions are smoothed out, discriminating among local peaks and valleys of gray intensity, as local tribes for teenagers to hang out with.



Figure 6. Iteration of the simple cellular automata "Look like my group and differentiate myself from other groups"

Figure 7 shows an image and the result after the initial "teenagers" cellular automata. Conservative agents are clearly defined as deep dark pixels but there is a huge range of liberal agents that go from dark to light distributed in more or less irregular groups.



Figure 7. Results after the initial "teenagers" cellular automata

To help them take a decision, we use local Otsu thresholding, add regional maxima and then apply simple image opening. The centers of dilated local maxima of the distance to zero transform shows regions of liberal concentration, true gravel. We chose circles around those points with the distance to nearest zero as the radio, and make those circles grow over the binarized image in order to determine true gravel regions. Similarly, we use the complements to determine true asphalt regions. The rest of the image are those agents that did not take a final decision during its youth, so they will make up their minds at a mature age, in the next stage of the society.



Figure 8. Classification of the pixels in three states

5. A Society of mature agents

A mature agent recognizes that his own class is a matter of probabilities. Indeed, for a given image processed in the teenager society, it is easy to estimate the gray level distribution for each class of pixel. Figure 9 shows the distribution corresponding to Figure 8.



Figure 9. Global distribution of the gray level for each class of pixels

Figure 9 also shows that its own gray level is not enough criteria for an agent to decide its class, so it needs to develop better cognitive capabilities:

Perception: Each agent is capable of perceiving the state of each pixel in its 31×31 neighborhood.

Attention: Among this 961 observed values, each agent pays more attention to closer data as it computes a 21-dim vector of features: its own gray level, the mean, standard deviation, maximum and minimum in a 3×3 neighborhood, the mean, standard deviation, maximum and minimum in a 7×7 neighborhood, the mean, standard deviation, maximum and minimum in a 15×15 neighborhood, and the mean, standard deviation, maximum and minimum in a 15×15 neighborhood, and the mean, standard deviation, maximum and minimum in a 31×31 neighborhood. These features of the neighborhood form a vector $\underline{d} \in \mathbb{R}^{17}$, which becomes the state of the agent, as it represents the perceived and processed information. However, since this data is highly redundant, each agent extracts the three principal components, giving a 3-dimensional state vector (Figure 10).



Figure 10. Three principal components of the 21-dim feature vector

Intelligence: Besides being able to compute its vector state form the perceived pixel values, the agent keeps a model of the world to help it understand its perception and decide how to act in its world. The world is interpreted as a mixture of Gaussians, where the state \underline{d} obey to one of two different Gaussian distributions, one for asphalt pixels with mean vector $\underline{\mu}_a$ and covariance matrix Σ_a , and another one for gravel pixels with mean vector $\underline{\mu}_g$ and covariance matrix Σ_g . The iterative estimation of these parameters is the individual learning process that leads to social learning and the emergency of an acceptable

segmentation. The agent is rational in the sense that, given the perceptions, it tries to maximize the likelihood by adapting its beliefs.

Memory: Each agent keeps track of its world model through the parameters $\underline{\mu}_a$, $\underline{\Sigma}_a$, $\underline{\mu}_g$ and $\underline{\Sigma}_g$. Indeed, these parameters are the code that summarizes what the agent has learned so far.

Action: Each agent is capable of increasing or decreasing its own gray level according to its perceptions and its model of the world. At the n^{th} iteration, it computes its probability of being gravel, $p_s(n)$, and adds to its own gray level the quantity $\beta_n \cdot (p_s(n)-0.5)$, where β_n is a learning rate parameter that decreases with time. By changing their gray value, agents act on its world according to their beliefs.

Language: The agents communicate indirectly among them stigmergically through their own action, since increasing or decreasing its gray level affects the environment (the perceptions) of the neighbor agents. But they also are capable of communicating their own state to each of its 31×31 neighbors in a single broadcast message. This is a local communication capability for a limited amount of information (that required to compute the next estimate of μ_s , μ_a , Σ_g , Σ_a and a_g). However, this iterated interaction eventually propagates the information over the whole image, given rise to the emergent segmentation of gravel and asphalt.

Now we describe the probabilistic rational behavior of the agents according to their cognitive capabilities. On the space of N = 3 principal components, the agents use a Gaussian mixture model (GMM). The fundamental idea is to assume that the probability density function of the features has the following form:

$$f\left(\underline{d} \middle| \underline{\mu}_{a}, \Sigma_{a}, \underline{\mu}_{g}, \Sigma_{g}, a_{g}\right) = a_{g} f_{g}\left(\underline{d} \middle| \underline{\mu}_{g}, \Sigma_{g}\right) + (1 - a_{g}) f_{a}\left(\underline{d} \middle| \underline{\mu}_{a}, \Sigma_{a}\right)$$

where $\underline{d} \in \mathbb{R}^3$ is the vector of principal components and

$$0 \le a_{g} \le 1$$

$$f_{g}\left(\underline{d} \middle| \underline{\mu}_{g}, \Sigma_{g}\right) = \frac{1}{\sqrt{(2\pi)^{N} \left| \Sigma_{g} \right|}} \exp\left(-\frac{1}{2} \left(\underline{d} - \underline{\mu}_{g}\right)^{T} \Sigma_{g}^{-1} \left(\underline{d} - \underline{\mu}_{g}\right)\right)$$

$$f_{a}\left(\underline{d} \middle| \underline{\mu}_{a}, \Sigma_{a}\right) = \frac{1}{\sqrt{(2\pi)^{N} \left| \Sigma_{a} \right|}} \exp\left(-\frac{1}{2} \left(\underline{d} - \underline{\mu}_{a}\right)^{T} \Sigma_{a}^{-1} \left(\underline{d} - \underline{\mu}_{a}\right)\right)$$

 $\underline{\mu}_{g}$ and $\underline{\mu}_{a}$ are also vectors in \mathbb{R}^{3} that correspond to the expected values of the features in the gravel and asphalt pixels, respectively. Σ_{g} and Σ_{a} are matrices in $\mathbb{R}^{3\times3}$ that correspond to the correlation matrices of the features for the asphalt and gravel pixels, respectively. The agent estimates $\underline{\mu}_{g}$, $\underline{\mu}_{a}$, Σ_{g} , Σ_{a} and a_{g} through a simple procedure: The 31×31 neighborhood is segmented into a binary image I_{S} through teenager cellular automata, and the estimations are obtained averaging over each kind of pixel:

$$\begin{split} a_g &= \frac{1}{N_p} \sum_{(x,y)} I_s(x,y) \\ \mu_g &= \frac{1}{N_p \cdot a_g} \sum_{(x,y)} I_s(x,y) \underline{d}(x,y) \\ \mu_a &= \frac{1}{N_p \cdot (1-a_g)} \sum_{(x,y)} \left(1 - I_s(x,y) \right) \underline{d}(x,y) \\ \Sigma_g &= \frac{1}{N_p \cdot a_g} \sum_{(x,y)} \left(\underline{d}(x,y) - \mu_g \right) \cdot \left(\underline{d}(x,y) - \mu_g \right)^T \cdot I_s(x,y) \\ \Sigma_a &= \frac{1}{N_p \cdot (1-a_g)} \sum_{(x,y)} \left(\underline{d}(x,y) - \mu_a \right) \cdot \left(\underline{d}(x,y) - \mu_a \right)^T \left(1 - I_s(x,y) \right) \end{split}$$

where $N_P = 961$ is the total number of pixels in the neighborhood, (x,y)

runs over the 31×31 neighborhood, and $\underline{d}(x,y)$ is the state of the cell in the position (x,y) of the neighborhood. Based on these estimates of $\underline{\mu}_{g}$, $\underline{\mu}_{a}$, Σ_{g} , Σ_{a} and a_{g} , the cell computes the probability of being gravel for each neighbor pixel using Bayes' rule:

$$P(G|\underline{d}(x,y)) = \frac{a_g f_g(\underline{d}(x,y)|\mu_g, \Sigma_g)}{a_g f_g(\underline{d}(x,y)|\mu_g, \Sigma_g) + (1-a_g) f_a(\underline{d}(x,y)|\mu_a, \Sigma_a)}$$
(1)

where (x, y) runs over all pixels in its neighborhood. This probability goes through one step of refinement for re-estimating the parameters:

$$\begin{split} &a_{g} = \frac{1}{N_{P}} \sum_{(x,y)} P\left(G | \underline{d}(x, y)\right) \\ &\mu_{g} = \frac{1}{N_{P} \cdot a_{g}} \sum_{(x,y)} \underline{d}(x, y) P\left(G | \underline{d}(x, y)\right) \\ &\mu_{a} = \frac{1}{N_{P} \cdot (1 - a_{g})} \sum_{(x,y)} \underline{d}(x, y) \left(1 - P\left(G | \underline{d}(x, y)\right)\right) \\ &\Sigma_{g} = \frac{1}{N_{P} \cdot a_{g}} \sum_{(x,y)} \left(\underline{d}(x, y) - \mu_{g}\right) \cdot \left(\underline{d}(x, y) - \mu_{g}\right)^{T} P\left(G | \underline{d}(x, y)\right) \\ &\Sigma_{a} = \frac{1}{NP \cdot (1 - a_{g})} \sum_{(x,y)} \left(\underline{d}(x, y) - \mu_{a}\right) \cdot \left(\underline{d}(x, y) - \mu_{a}\right)^{T} \left(1 - P\left(G | \underline{d}(x, y)\right)\right) \end{split}$$

Finally, these new parameters are used to re-estimate the probability of the pixel to be gravel, as in equation (1). This is just one step of the EM algorithm [16], but the agent does not iterate it until convergence, because this is simply an intermediate opinion to be shared with the neighborhood, in order to emerge a global opinion, the segmented image. Indeed, this probability represents the belief of the agent about its own classification. It tries to enforce this belief on its neighborhood by adding to its own gray level the quantity $\beta \cdot (P(G|\underline{d}(x,y)) - 0.5)$, where β is a learning rate parameter that can be decremented with time.

This process completes the perception/action cycle of each cognitive agent, which becomes a single step of the mature cellular automata. This cellular automata algorithm is repeated until convergence. Then, as the last step, we repeat the post-processing of the teenager cellular automata: local Otsu thresholding, regional maxima, image opening, distance transform, and region growing.

6. SEGMENTATION RESULTS

Figure 11 shows the evolution of $P(G|\underline{d}(x,y))$, the local probability of each pixel to be gravel, during 15 steps of the cellular automata, which bring the automata close to equilibrium. Once the pixels have agreed on the probability of being gravel, the simple post-processing is performed, leading to the result shown in Figure 12.

We have used several segmentation methods, not pretending to be exhaustive, but only to have something to compare with. As shown in Figure 13, we use an adaptive Otsu's method, a k-means approach with the same parameters in \mathbb{R}^{17} , a neural network with these 17 inputs trained with the manually segmented images, and a GMM/EM method. Recently we submitted a paper with a "committee of experts" method in which the four methods of Figure 13 are added and postprocessed with several heuristics [18]. Classification results are not as satisfactory as those of the two phase cellular automata, and time processing is several times bigger than that of the two phase cellular automata.



Figure 11. Evolution of social learning among cognitive agents



Figure 12. Final classification through cognitive social learning cellular automata



Figure 13. Segmentation by (a) adaptive Otsu, (b) 2-means on \mathbb{R}^3 , (c) Neural network and (d) GMM/EM

As a final comparison, when the iteration with several graduate students starts with the output of the cognitive social learning cellular automata, the number of iterations and modifications is drastically reduced: The performance of our cognitive classifier got close to the human classifier.

7. CONCLUSIONS

Cognitive cellular automata is an interesting approach to complex systems engineering, since it can be applied as a mathematical model of a great number of complex systems in science and engineering. In this paper we used a simple model of social learning and opinion formation as an inspiration for granular segmentation. Using two stages of cognitive cellular automata, one resembling teenager's tribe formation and another resembling mature opinion formation, we obtained a good asphalt/gravel classifier in pavement images. The results are better than traditional segmentation algorithms and the method requires less computation time. As a future work, we will consider opinion changes, so that we can track variations of class in dynamical distributed systems. Such changes could be used as inspiration for many engineering problems, such as collaborative access in cognitive radio networks.

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