Effects of Topology and Mobility in Bio-Inspired Synchronization of Mobile Ad Hoc Networks

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Spontaneous Synchronization

• Spontaneous synchronization occurs ubiquitously in nature (from particles, to atoms, to molecules, to cells, to insects, to humans, to planets, to galaxies, ...)

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Basic model


Each oscillator follows a simple law of “Integrate and fire”

Then they interact through a simple rule: When an oscillator fires, neighbors state advances a small quantity $\varepsilon$

$$ x_i(t) = 1 \Rightarrow \begin{cases} 
  x_i(t^+) = 0 \\
  x_j(t^+) = \min(1, x_j(t^-) + \varepsilon) \quad j \text{ neighbor of } i, j \neq i 
\end{cases} $$
Charging functions

At instant $t$, an oscilador is in a fraction $\phi$ of its phase,

$$x(t) = f(\phi(t)) = \frac{1}{b} \log \left( 1 + (e^b - 1)\phi \right)$$

- When the oscillator is in state $x(t)$, it is in a fraction $\phi$ of its phase,

$$\phi(t) = g(x(t)) = g\left(f(\phi(t))\right) = \frac{e^{bx} - 1}{e^b - 1}$$
Two coupled oscillators

\[ x(t) \]

\[ \phi(t) \]
Two coupled oscillators

$\phi(t)$

$x(t)$

A

B

$\varepsilon$

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Two coupled oscillators

\[ x(t) \]

\[ \phi(t) \]

\( \varepsilon \)

B
A

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Let us put an stroboscope immediately after A fires
Two coupled oscillators

- After A fires, A’s phase is zero and B’s phase is $\phi$.
- B will fire when its phase advances $1 - \phi$. By then, A will be in $x_A = f(1 - \phi)$.
- Then, B returns to zero and A jumps to $x_A = \min(1, \varepsilon + f(1 - \phi))$.
- If $x_A = 1$, we achieved synchronism.
- otherwise, $x_A = \varepsilon + f(1 - \phi) < 1$ and A’s phase is $h(\phi) = g(\varepsilon + f(1 - \phi))$.
- In the next round, if they have not achieved synch, their phases will be $(0, h(h(\phi)))$.
- **Return map**: phase of B the next time A fires, if it was at phase $\phi$ at the previous firing of A

$$R(\phi) = h(h(\phi)) = g\left(\varepsilon + f\left(1 - g\left(\varepsilon + f\left(1 - \phi\right)\right)\right)\right)$$
Two coupled oscillators

The return map has a unique fixed point, 

\[ \phi^* = \frac{e^{b(1+\varepsilon)} - 1}{(e^b - 1)(e^{b\varepsilon} - 1)} \]

Which is a repeller

\[ \varepsilon = 0.05, \ b = 1 \]

\[ \varepsilon = 0.05, \ b = 3 \]

\[ \varepsilon = 0.05, \ b = 10 \]

\[ \varepsilon = 0.10, \ b = 1 \]

\[ \varepsilon = 0.10, \ b = 3 \]

\[ \varepsilon = 0.10, \ b = 10 \]
Synchronization is unavoidable

Independent Oscillators

\( b = 3, \ \varepsilon = 0.0 \)

Weakly coupled oscillators

\( b = 3, \ \varepsilon = 0.05 \)
Mobile Ad Hoc Networks
Mobile Ad Hoc Networks
Why synchronization is important in mobile ad hoc networks

At physical layer
  – Frequency Hopping

• At multiple access layer
  – Packet scheduling

• At network layer
  – Temporary assignment of transmission resources

• At Transport Layer
  – Synchronization of transmission patterns for maximum utilization of resources

• At application layer
  – Consistent ordering of events in sensor networks
MANET Bio-inspired Synchronization

$b=3$ and $\varepsilon=0.006$
MANET Bio-inspired Synchronization

Average state among seven nodes, another view of synchronization

$b=3$ and $\varepsilon=0.006$
For Synchronization the first topology requires 17 pulses, the second topology requires 75.
Effects Of topology

100 nodes in a regular topology
Effects of topology

Average state among the 100 nodes

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Effects of topology

\[(\phi_1, 0, \phi_3) \rightarrow (\phi_1', 0, \phi_3')\]

\[\phi_1' = g(\varepsilon + f[1+\phi_1-\phi_3-g(\varepsilon + f[\phi_1-\phi_3+g(\varepsilon + f[1-\phi_1])])])\]

\[\phi_3' = g(\varepsilon + f[1-g(\varepsilon + f[\phi_1-\phi_3+g(\varepsilon + f[1-\phi_1])]))]\]

The only equilibrium point of this 4-dimensional return map is unstable: As long as there are no partitions, a network will get synchronized.

All trajectories in the phase space lead to synchronization.
Effects of Mobility

74 pulses for synchronization for non-moving nodes

Mobility accelerates the dissemination of synch information

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Effects of Mobility

A Dynamic partitioned network

Through mobility, the clusters of a partitioned network get synchronized.
Effects Of mobility

Three nodes interacting indirectly through mobility

Region where the given return map holds, for \( b=3 \) and \( \varepsilon=0.06 \), where \( h(\alpha) \equiv g(\varepsilon+f(\alpha)) \)

\[ R \left( \begin{bmatrix} \phi_1 \\ \phi_3 \end{bmatrix} \right) = \begin{bmatrix} h(1-h(1-\phi_1)) + h(1-\phi_1) + \phi_1 - \phi_3 - h(h(1-\phi_1) + \phi_1 - \phi_3) \\ h(1-h(\phi_1 - \phi_3 + h(1-\phi_1))) \end{bmatrix} \]
# Effects of Mobility

<table>
<thead>
<tr>
<th>Time</th>
<th>Phases</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>((\phi_1, 0, \phi_3))</td>
<td>(n_2) has just fired, leaving the given phases, and moves near (n_1), which is the next node to fire</td>
</tr>
<tr>
<td>1-(\phi_1)</td>
<td>((0, h(1-\phi_1), 1-(\phi_1-\phi_3)))</td>
<td>(n_1) has just fired, affecting (n_2). Next node to fire is (n_3).</td>
</tr>
<tr>
<td>1-(\phi_3)</td>
<td>((\phi_1-\phi_3, \phi_2, 0)) where (\phi_2=h(1-\phi_1)+\phi_1-\phi_3)</td>
<td>(n_3) has just fired, affecting none. Next node to fire is (n_2).</td>
</tr>
<tr>
<td>2-((\phi_2+\phi_3))</td>
<td>((\phi_1', 0, 1-\phi_2)) where (\phi_1'=h(1-h(1-\phi_1)))</td>
<td>(n_2) has just fired, affecting (n_1), and moves near (n_3). Next node to fire is (n_1).</td>
</tr>
<tr>
<td>3-((\phi_1'+\phi_2+\phi_3))</td>
<td>((0,1-\phi_1', 2-(\phi_1'+\phi_2)))</td>
<td>(n_1) has just fired, affecting none. Next node to fire is (n_3).</td>
</tr>
<tr>
<td>2-(\phi_3)</td>
<td>((\phi_1'+\phi_2-1, h(\phi_2), 0))</td>
<td>(n_3) has just fired, affecting (n_2). Next node to fire is (n_2).</td>
</tr>
<tr>
<td>3-(\phi_3-h(\phi_2))</td>
<td>((\phi_1'+\phi_2-h(\phi_2),0, h(1-h(\phi_2)))</td>
<td>(n_2) has just fired, leaving the given phases, and moves near (n_1). This is the given return map.</td>
</tr>
</tbody>
</table>
Effects of Mobility

Phase space trajectories of nodes $n_1$ and $n_3$, at firing instants of node $n_2$, immediately before moving to the first position
Conclusions

• Firefly synchronization in wireless ad hoc networks does not need to rely in a completely interconnected topology.

• Even under the case of partitioned networks, mobility can establish positive feedback interactions among the nodes, so synchronization can emerge easily in temporally partitioned mobile ad hoc networks.

• To accelerate the synchronization process, the interactions among nodes must be encouraged, either by forming dense clusters or by increasing mobility.

• (By the way, This is an ongoing research work in which we want to keep synchronism within a MANET to coordinate bandwidth estimation procedures, transmission schedule, and resource reservation schemes).
The only risk is wanting to stay

Gracias! (Thank you 😊)

Questions?

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