Effects of Topology and Mobility in Bio-Inspired Synchronization of Mobile Ad Hoc Networks

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Abstract—The interest on firefly approaches to the problem of synchronizing the nodes of a wireless ad hoc communication network is rising, because of its efficiency and efficacy. In this paper we show that it is enough to have an indirect interaction among nodes, either spatially (through multi-hop paths), or temporally (through mobility) to achieve synchronism among them. As the interactions among nodes increases, global synchronization emerges faster.

Keywords—emergent phenomena, self-organization, firefly synchronization, MANET

I. INTRODUCTION

Mobile ad hoc wireless networks (MANETs) are characterized by the absence of communication infrastructure and by the mobility of the hosts, which also act as routers [1]. In such a distributed environment, self-organization is a key concept for network configuration and operation [2]. Bio-inspired self-organization principles have been successfully applied in this and many other fields, either indirectly as meta-heuristics for solving optimization problems within the network (like genetic algorithms for finding optimal strategies in game theoretic trust models [3], for example), or directly as distributed algorithms for implementing a network function (like ant colony based routing for MANETs [4], for example). Time synchronization is one of the network functions that require a new self-organization approach for MANETs, since it has been solved in a highly centralized way in infrastructure networks, with some kind of master clock transmitting time information (see [5], for example). This function is very important for many control mechanisms at different layers in wireless networks, from frequency hopping in spread spectrum radio, to packet scheduling in routing and flow control, to event ordering in sensor applications.

In this paper we report some experimental results on wireless ad hoc network synchronization, based on coupled oscillators and inspired in the behavior of some species of Asian fireflies, which can synchronize their blinking [6]. Buck [7] noticed that each firefly alone keeps a stable rhythm, which can be adjusted by a visual stimulus, like the blinking of neighbor fireflies, so their synchronization becomes a cooperative emergent phenomenon. Then, based on this principle, Mirollo and Strogatz [8] developed a mathematical model of pulse coupled oscillators that showed the inevitability of synchronization in a completely interconnected network of fireflies.

In order to use this firefly principle in MANET synchronization, there are (at least) three issues to solve: (1) not all nodes detect the blinking of each other, (2) the coupling pulses arrive with random delays, and (3) mobility changes the coupling relations among nodes. Lucarelli and Wang [9] showed that this type of synchronization can evolve in a not fully connected network. Later, Tyrrell et.al. [10][11] took into account that, in wireless ad hoc networks, there is no such pulse-coupling, due to multiple access delays, transmission delays, and packet decoding delays.

In this paper we consider the case of non-completely interconnected networks and notice that, as long as the network is not partitioned, synchronization will finally emerge. Furthermore, if the network gets partitioned but some mobile nodes travel among partitions, the whole network will eventually synchronize. The contributions include the study of effects of mobility and the direct extension of Mirollo-Strogatz analytical model, which leads to the conclusion that any indirect interaction among nodes, either spatially (through multi-hop paths), or temporally (through mobility), will suffice for synchronization to emerge spontaneously.

The paper is organized as follows. In section II we describe the model of pulse-coupled oscillators, which explain firefly synchronization. In section III we present the simulation and analytical results on non-completely interconnected topologies and partitioned networks with mobility. Section IV concludes the paper.

II. PULSE-COUPLED OSCILLATORS

As already mentioned, [8] reports a mathematical model to explain firefly synchronization. Each firefly is an oscillator that interacts with other fireflies through simple local rules, from which global synchronization emerges. The state of the oscillator, \( x(t) \), increases from 0 to 1 during a cycle and, after reaching the value 1, the firefly blinks and the state is reset to zero immediately. If \( \phi(t) \) is a phase variable, in the range \([0, 1]\), then the state is a function of the phase, \( x(t) = f(\phi(t)) \), with \( df/dt = 1/T \), where \( f([0,1] \rightarrow [0,1]) \) is a function with \( f' > 0, f' < 0, f(0) = 0 \) and \( f(1) = 1 \). With these properties, there exist an inverse function \( g = f' \), so that \( \phi(t) = g(x(t)) \). Now consider two fireflies, \( A \) and \( B \), which interact in the following simple way: when \( x_A(t) \) reaches 1 and fires, then \( x_A(t') \) becomes \( \min(1, x_A(t)+\epsilon) \) and \( x_A(t') \) becomes 0. Similarly, when \( x_B(t) \) reaches 1 and fires, then \( x_B(t') \) becomes \( \min(1, x_B(t)+\epsilon) \) and \( x_B(t') \) becomes 0. The coupling parameter, \( \epsilon \), is just the pull up between fireflies. Fig. 1 shows the state of two fireflies under the functions.
\[ x(t) = f(\phi(t)) = \frac{1}{b} \log \left( 1 + (e^b - 1) \phi \right) \]

\[ \phi(t) = g(x(t)) = \frac{e^{hx} - 1}{e^b - 1} \]

for the case \( b=3 \) and \( \varepsilon = 0.05 \), beginning with random phases. After six cycles, fireflies achieve synchronicity and remain synchronized thenceforth.

Fig. 1. Synchronization between two fireflies, modeled as pulse-coupled oscillators

It is easy to verify that synchronization is unavoidable. Immediately after \( A \) fires, its phase is \( \phi_j = 0 \) and the phase of \( B \) is some number \( \phi \). So, \( B \) will fire when its phase increases \( 1 - \phi \). During that time, \( A \) has moved to \( x_A = f(1 - \phi) \) (we are assuming \( d\phi/dt \) is the same for both fireflies). Immediately after that, \( B \) comes back to 0 and \( A \) jumps to \( \min(1, f(1-\phi) + \varepsilon) \). If \( x_A = 1 \), we achieved synchronism; otherwise, \( x_A = \varepsilon + f(1 - \phi) < 1 \) and its phase is \( h(\phi) = g(\varepsilon + f(1 - \phi)) \). In the second case, the fireflies went from \((0, \phi) \) to \((h(\phi), 0) \) and, in the next round, they will go to \((0, h(h(\phi))) \) (if they do not synchronize). Let us call \( R(\phi) = h(h(\phi)) \) the return map, i.e., the phase of \( B \) at the next firing of \( A \) when its phase was \( \phi \) at the previous firing of \( A \) (if they did not get synchronized),

\[ R(\phi) = h(g(\varepsilon + f(1 - \phi))) = g(\varepsilon + f(1 - g(\varepsilon + f(1 - \phi)))) \]

Under the function given in Equation (1), the fixed point of \( R(\phi) \), i.e., the phase that achieves \( \phi^* = R(\phi^*) \), is

\[ \phi^* = \frac{e^{h(1+\varepsilon)} - 1}{(e^b - 1)(e^{h\varepsilon} - 1)} \]

which is a repelling fixed point as long as \( b>0 \) and \( \varepsilon>0 \) (Fig. 2), so the synchronization is unavoidable.

Mirollo and Strogatz [8] also proved that synchronization is unavoidable in a group of \( n \) completely interconnected fireflies. Just enumerate them from 0 to \( n-1 \) according to their phases, and consider an instant of time immediately after a firing,

\[ 0 = \phi_0 < \phi_1 < \cdots < \phi_{n-1} < 1 \]

A simpler way to see this phenomenon under many fireflies is by plotting the average state among them, as shown in Fig. 4.

Fig. 2. The return map and its unstable fixed point

Next firing firefly is \( n-1 \), which become the 0th firefly with \( \phi_n = 0 \), while the previously \((n-1)^{th} \) firefly becomes the \( j^{th} \), for \( j=1,2,\ldots,n-1 \). So, after firing, the next phase states are

\[ 0, (g(f(1-\phi_{j+1}) + \varepsilon), g(f(1-\phi_{j+1}) + \varepsilon), \ldots g(f(1-\phi_{j+1}) + \varepsilon)) \]

where, possibly, some of the \( f(\phi_{j+1} + \varepsilon) \) are bigger than \( 1-\varepsilon \), in which case those fireflies complete their cycles and are “absorbed” by the previous firing. These absorptions create groups of fireflies that blink together, where the pulse intensity is proportional to the number of fireflies in the group. This is a positive feedback process where the biggest group gets bigger so, again, synchronization is unavoidable, as shown in Fig. 3 for seven fireflies with \( b=3 \) and \( \varepsilon=0.006 \). They are green, red, magenta, purple, black, blue and yellow. At the first cycle, the red firefly absorbs the green one and the purple firefly absorbs the magenta one; at the third cycle, the black firefly joins the red-green group, and the whole group becomes black; at the fifth cycle, the blue firefly joins the magenta-purple group, and the whole group remains purple; At this point there are two groups of three fireflies each (green-red-black and magenta-blue-purple), and an isolated yellow firefly. At the seventh cycle the yellow firefly joins the purple group and, after the eighth cycle, the seven fireflies blink together.

Fig. 3. Synchronization between seven fireflies
III. EFFECTS OF TOPOLOGY AND MOBILITY IN
Synchronizing a MANET

Now we consider one of the many difficulties in MANETs, with respect to the encouraging results of Fig. 4: Complete interconnection is a rare condition.

Consider four nodes in a completely interconnected topology (Fig. 5(a)) and in a row (Fig. 5(b)). The average state of the four nodes, for the same random initial phases, under the two different topologies, is plotted in Fig. 6. It is easy to notice the higher synchronization delay in the second topology: while the first topology requires 17 pulses, the second topology requires 75.

![Fig. 5. Two topologies in a four-node MANET](image)

![Fig. 6. Average state among four nodes in the two topologies of Fig. 5](image)

It is interesting to notice that, anyway, the network will eventually get synchronized. Indeed, consider the 100-node network of Fig. 7, for which the average state is shown in Fig. 8. Looking at them like flashing fireflies, after blinking randomly for a while, some geographically localized groups begin to synchronize, and, after a small period in which there are flashing waves over the network, they start all blinking together (after 74 pulses).

This phenomenon can be explained by the fact that any non-partitioned topology is formed by overlapped maximal cliques (completely interconnected sub-graphs not contained in a bigger completely interconnected sub-graph). Since each maximal clique will unavoidably synchronize, the coupling between maximal cliques will make them synchronize, too. Consequently, the lack of complete interconnection delays the synchronization of the whole network, but do not avoid them, as long as the network is not partitioned.

![Fig. 7. 100 nodes in a regular topology](image)

![Fig. 8. Average state among the 100 nodes of Fig. 7](image)

Consider, for example, the topology of Fig. 9, with three nodes \( n_1, n_2 \) and \( n_3 \) and two maximal cliques \( (m_1, m_2) \). Let us look at the system just after \( n_2 \) has fired and, without loss of generality, let us call \( n_1 \) the next node to fire. So, the phases at that time are \( (\phi_1, 0, \phi_3) \), with \( \phi_1 > \phi_3 \). After 1-\( \phi_1 \) periods, \( n_1 \) will fire and the new phases are \( (0, \phi_2, \phi_1 + 1 - \phi_1) \), where \( \phi_2 = g(\epsilon + f(1 - \phi_1)) \). If \( \phi_2 > \phi_1 + 1 - \phi_1 \), the next node to fire is \( n_2 \) and it will absorb \( n_3 \), so \( m_2 \) will get synchronized. Otherwise, the next node to fire is \( n_1 \) and, after it happens, at time 1-\( \phi_1 \), the next phase state will be \( (\phi_1, \phi_2, 0) \), where \( \phi_2' = g(\epsilon + f(\phi_1 + \phi_2)) \).
Next question is how mobility affects synchronization. To begin finding an answer, the nodes of Fig. 7 will change positions with randomly selected neighbors at randomly selected times. For the same initial phases, we record the time until the first synchronized pulse, and plot it against the velocity of the nodes, averaging over 50 random initial phases (Fig. 12). Clearly, when a node moves, it transfers synchronization information among groups within the network, increasing the effect of overlapping maximal cliques. Fig. 12 shows how, as the mobility increases, the global synchronization comes faster.

Indeed, even under a partitioned network topology, mobility can produce temporal couplings among clusters, from which global synchronization can emerge. Just as an example, consider the network of Fig. 13, partitioned in two clusters, where the fourth node changes periodically between two positions, so it belongs to each of the two clusters intermittently. At first position, node 4th influences, and gets positions, so it belongs to each of the two clusters. Consequently, although the network topology allows a path between every pair of nodes, synchronization is unavoidable.

Fig. 13. A Dynamic partitioned network

(a) First position

(b) Second position

Fig. 12. Effect of mobility in synchronization convergence

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Depending on the particular point on the plane (node compute the return map for each cluster at firing epochs of can achieve synchronism? As we obtained Eq. (5), we can

\[
\phi_3 = \phi_1 + \phi_2 - \phi_3
\]

where \( \phi \) leaving the phases (\( \phi_1, 0, \phi_3 \)), and moves to the first position. Depending on the particular point on the plane (\( \phi_1, \phi_3 \)), there could be different return maps for each of the phases of \( n_1 \) and \( n_2 \). For example, when (\( \phi_1, \phi_3 \)) lies within the pink area of Fig. 16, the return map is

\[
R \begin{bmatrix} \phi_1 \\ \phi_3 \end{bmatrix} = \begin{bmatrix} h(1-h(1-\phi_1)) + h(1-\phi_3) + h(1-h(1-\phi_1) + \phi_3 - h(1-h(1-\phi_1) + \phi_3 - h(1-h(1-\phi_1)) \\ h(1-h(1-\phi_1) + \phi_3 - h(1-h(1-\phi_1)) \end{bmatrix} (6)
\]

where \( h(\phi) = g(e^f(\phi)) \).

Just as an example, when \( \phi_1 > \phi_3 \) within that region, the sequence of states is as shown in Table I.

<table>
<thead>
<tr>
<th>Time</th>
<th>Phases</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(( \phi_1, 0, \phi_3 ))</td>
<td>( n_2 ) has just fired, leaving the given phases, and moves near ( n_1 ), which is the next node to fire</td>
</tr>
<tr>
<td>1-( \phi_1 )</td>
<td>(0, ( h(1-\phi_1), 1-(\phi_2-\phi_3) ))</td>
<td>( n_1 ) has just fired, affecting ( n_2 ). Next node to fire is ( n_1 ).</td>
</tr>
<tr>
<td>1-( \phi_2 )</td>
<td>(( \phi_3-\phi_1, \phi_3, 0 )) where ( \phi_1 = h(1-\phi_1) + \phi_2 - \phi_3 )</td>
<td>( n_3 ) has just fired, affecting none. Next node to fire is ( n_2 ).</td>
</tr>
<tr>
<td>2-( \phi_2+\phi_3 )</td>
<td>(( \phi_3, 0, 1-\phi_2 )) where ( \phi_2 = h(1-h(1-\phi_1)) )</td>
<td>( n_2 ) has just fired, affecting ( n_1 ). Next node to fire is ( n_1 ).</td>
</tr>
<tr>
<td>3-( \phi_3+\phi_3+\phi_1 )</td>
<td>(0, 1-( \phi_3 ), 2-( \phi_3+\phi_2 ))</td>
<td>( n_1 ) has just fired, affecting none. Next node to fire is ( n_1 ).</td>
</tr>
<tr>
<td>2-( \phi_3 )</td>
<td>(( \phi_3, -\phi_3 - 1, h(\phi), 0 ))</td>
<td>( n_3 ) has just fired, affecting ( n_2 ). Next node to fire is ( n_2 ).</td>
</tr>
<tr>
<td>3-( \phi_3-h(\phi) )</td>
<td>(( \phi_3 + \phi_3 - h(\phi), 0, h(1-h(\phi)) ))</td>
<td>( n_2 ) has just fired, leaving the given phases, and moves near ( n_1 ). This is the return map of Eq. (6).</td>
</tr>
</tbody>
</table>

As before, the corresponding equilibrium points are unstable, so the system will eventually synchronize, similarly to Fig. 2 and Fig. 10. However, due to the intermittent interaction in time, the trajectories in phase space are not as smooth as those in Fig. 11, but finally they will arrive at (1,0), where synchronization has been achieved (See Fig. 17).

**IV. Conclusions**

Firefly synchronization in wireless ad hoc networks does not need to rely in a completely interconnected topology. Even under the case of partitioned networks, mobility can establish positive feedback interactions among the nodes, so synchronization can emerge easily in temporally partitioned
mobile ad hoc networks. To accelerate the synchronization process, the interactions among nodes must be encouraged either by forming dense clusters or by increasing mobility.

This is an ongoing research work in which we want to keep synchronism within a MANET to coordinate bandwidth estimation procedures, transmission schedule, and resource reservation schemes. Right now we are evaluating the effects of transmission impairments to adjust the synchronization protocol in order to apply them in the mentioned applications.

REFERENCES